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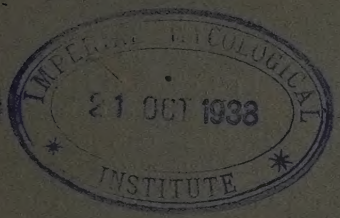

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MATHÉMATIQUES

SUR LA THÉORIE DE LA REPRÉSENTATION CONFORME DES DOMAINES DOUBLEMENT CONNEXES

Par G. J. KHAJALIA

(Présenté par I. M. Vinogradow, de l'Académie, le 3.V.1938)

1. Soit D un domaine jordanien fini doublement connexe situé dans le plan de la variable complexe z et soient c_1 et c_2 ses frontières extérieure et intérieure.

Considérons dans le domaine D un champ de courbes telles que l'une des extrémités de chaque courbe du champ est située sur c_1 et l'autre sur c_2.

Soient

$$\left.\begin{aligned} x &= f_1(s, \varphi), \qquad 0 \leqslant \varphi \leqslant 2\pi, \\ y &= f_2(s, \varphi), \qquad s_1(\varphi) < s < s_2(\varphi) \end{aligned}\right\} \tag{1}$$

les équations des courbes de notre champ, φ étant le paramètre qui définit la courbe du champ et s le paramètre désignant la longueur de l'arc de la courbe.

Nous allons supposer dans la suite que les fonctions $f_1(s, \varphi)$ et $f_2(s, \varphi)$ sont des fonctions continues ainsi que leurs dérivées partielles du premier ordre dans tout le domaine où varient s et φ et que d'ailleurs le déterminant fonctionnel

$$\frac{D(x, y)}{D(s, \varphi)} = J(s, \varphi)$$

est non nul.

Théorème I. *En faisant la représentation conforme $w = f(z)$ du domaine doublement connexe D sur l'anneau $1 < |w| < R$, on a pour chaque champ* (1) *possédant les propriétés indiquées de l'inégalité*

$$\int_0^{2\pi} \frac{d\varphi}{\int_{s_1(\varphi)}^{s_2(\varphi)} \frac{ds}{J(s,\varphi)}} \leqslant \frac{2\pi}{\operatorname{Log} R}, \tag{2}$$

le signe d'égalité n'ayant lieu que dans le cas où les lignes du champ forment avec le champ des lignes orthogonales un réseau isothermique, c'est-à-dire dans le cas où dans la représentation $w = f(z)$ il correspond aux courbes du champ (1) *des segments $1 < |w| < R$ sur les rayons du cercle $|w| < R$.*

Démonstration. En effet, supposons que dans la représentation $w=f(z)$ une courbe de la famille (1) pour $\varphi=\varphi_1$ ne se transforme pas en un segment sur un rayon du cercle. Donnons au paramètre φ dans les équations (1) les valeurs:

$$\varphi_1=\varphi_1^{(n)},\ \varphi_2^{(n)},\ \ldots,\ \varphi_n^{(n)},\ \varphi_j^{(n)}=\varphi_{j-1}^{(n)}+\frac{2\pi}{n},\ (j=2,3,\ \ldots\ n),$$

alors dans le domaine D on aura n courbes de la famille; elles divisent le domaine D en n domaines $D_j^{(n)}$ jouissant des propriétés:

1. Les domaines $D_j^{(n)}$ $(j=1,2,\ \ldots,\ n)$ sont deux à deux sans points communs.

2. La frontière de $D_j^{(n)}$ $(j=1,2,\ \ldots,\ n)$ est formée de deux arcs A_jB_j, $A_j'B_j'$ de courbes de la famille appartenant au domaine D et dont les extrémités A_j, A_j' sont sur c_2, B_j et B_j' sur c_1 et des arcs A_jA_j' et B_jB_j' des courbes c_1 et c_2.

Réalisons la représentation conforme des domaines $D_j^{(n)}$ $(j=1,2,\ \ldots,\ n)$ sur un certain rectangle $R_j^{(n)}$ de telle manière qu'aux points A_j, B_j, A_j' et B_j' correspondent les sommets du rectangle. En désignant par $b_j^{(n)}$ la longueur des côtés du rectangle qui correspondent aux arcs A_jB et $A_j'B_j'$ et par $a_j^{(n)}$ la longueur de chacun des autres côtés du même rectangle, on a en vertu d'un théorème de Grötzsch [1] l'inégalité:

$$\sum_1^n \frac{a_j^{(n)}}{b_j^{(n)}} \leqslant \frac{a_1^{(0)}}{b_1^{(0)}} < \frac{2\pi}{\operatorname{Log} R}. \tag{3}$$

Considérons le domaine $D_j^{(n)}$. Les équations des arcs A_jB_j et $A_{j+1}B_{j+1}$ seront

$$\left.\begin{aligned} x&=f_1(s,\varphi_j), \quad s_1(\varphi)<s<s_2(\varphi),\\ y&=f_2(s,\varphi_j), \quad \varphi=\varphi_j=\text{const}\end{aligned}\right\} \tag{4}$$

et

$$\left.\begin{aligned} x&=f_1(s,\varphi_j+\Delta\varphi), \quad s_1(\varphi)<s<s_2(\varphi_2),\\ y&=f_2(s,\varphi_j+\Delta\varphi).\end{aligned}\right\} \tag{5}$$

En considérant $\Delta\varphi$ comme infiniment petit, cherchons la valeur du rapport $\frac{a_j^{(n)}}{b_j^{(n)}}$ en négligeant les infiniment petits d'ordre supérieur.

À cet effet considérons sur la courbe (4)

$$x=f_1(s,\varphi_j),$$
$$y=f_2(s,\varphi_j),$$

le point $m=m(s_0,\varphi_j)$ et menons par ce point une normale mn jusqu'à l'intersection avec la ligne (5):

$$x=f_1(s,\varphi_j+\Delta\varphi),$$
$$y=f_2(s,\varphi_j+\Delta\varphi).$$

Supposons que dans la représentation conforme du domaine $D_j^{(n)}$ sur $R_j^{(n)}$ au point $m=m(s_0,\varphi_j)$ il correspond sur un côté de longueur $b_j^{(n)}$ du rectangle $R_j^{(n)}$ un point dont la distance du sommet de $R_j^{(n)}$ est égale à σ; σ croît avec s.

Considérons sur la courbe:

$$x = f_1(s, \varphi_j),$$
$$y = f_2(s, \varphi_j).$$

un point infiniment voisin du point m, $m' = m'(s_0 + \Delta s, \varphi_j)$, et menons par ce point aussi une normale $m'n'$ jusqu'à l'intersection avec la courbe:

$$x = f_1(s, \varphi_j + \Delta\varphi),$$
$$y = f_2(s, \varphi_j + \Delta\varphi).$$

Sur le côté de $R_j^{(n)}$ ayant la longueur $b_j^{(n)}$ il correspond à ce point un certain point $\sigma + \Delta\sigma$.

En posant $a_j^{(n)} = \Delta\varphi$, définissons respectivement $b_j^{(n)}$. À un rectangle élémentaire, ayant pour côtés $\Delta\varphi$, $\Delta\sigma$ et situé dans le rectangle $R_j^{(n)}$, il correspond, quand on néglige les infiniment petits d'ordre supérieur, un rectangle dont les longueurs des côtés sont Δs et $mn = \delta_n$.

En vertu de la représentation conforme on a:

$$\frac{\Delta\varphi}{\Delta\sigma} = \frac{\delta_n}{\Delta s}$$

ou bien

$$b_j = \sum \Delta\sigma = \sum \frac{\Delta\varphi}{\delta_n} \Delta s.$$

Calculons la longueur de la normale δ_n; nous aurons:

$$\delta_n = \frac{D(x,y)}{D(s,\varphi)} \Delta\varphi = J(s,\varphi)\, \Delta\varphi.$$

En passant à la limite pour $n \longrightarrow \infty$ on obtient finalement:

$$b_j^{(n)} = b_j = \int_{s_1(\varphi)}^{s_2(\varphi)} \frac{ds}{J(s,\varphi)}. \tag{6}$$

En substituant dans (3) au lieu de $a_j^{(n)}$ sa valeur $\Delta\varphi$ et au lieu de $b_j^{(n)}$ son expression (6) on a, en négligeant des valeurs infiniment petites avec $\Delta\varphi$, l'inégalité:

$$\sum_1^n \frac{\Delta\varphi}{\int_{s_1(\varphi)}^{s_2(\varphi)} \frac{ds}{J(s,\varphi)}} < \frac{a_1^{(0)}}{b_1^{(0)}} < \frac{2\pi}{\operatorname{Log} R},$$

et en passant à la limite pour $n \longrightarrow \infty$ on trouve finalement:

$$\int_0^{2\pi} \frac{d\varphi}{\int_{s_1(\varphi)}^{s_2(\varphi)} \frac{ds}{J(s,\varphi)}} < \frac{2\pi}{\operatorname{Log} R}.$$

Le théorème démontré est une généralisation du lemme cité de Grötzsch; il nous semble que dans diverses applications la forme que nous avons donnée à ce lemme présente cet avantage essentiel que le premier membre de l'inégalité fondamentale est défini immédiatement à partir des courbes du champ.

En particulier notre théorème permet de déterminer directement le nombre R

$$\frac{1}{2\pi}\operatorname{Log} R = \min \frac{1}{\frac{1}{2\pi}\int_0^{2\pi}\int_{s_1(\varphi)}^{s_2(\varphi)} \frac{ds}{J(s,\varphi)}\,d\varphi}.$$

Les courbes de la famille (1), réalisant ce minimum, forment avec le champ des courbes orthogonales un réseau isothermique, c'est-a-dire, définissent la représentation conforme du domaine D sur un anneau circulaire.

2. On obtient facilement du théorème démontré une autre propriété minimale de la fonction, réalisant la représentation conforme d'un domaine doublement connexe sur un anneau.

Théorème II. *Soit D un domaine doublement connexe et d'ailleurs arbitraire situé dans le plan de la variable complexe z; soit R le module du domaine D et soit $\{f(z)\}$ une famille de fonctions analytiques régulières dans le domaine D et jouissant des propriétés*:

1. *$f(z)$ est uniforme*;
2. $|f(z)| \geqslant 1$;
3. $\frac{1}{2\pi i}\int_c \frac{f'(z)}{f(z)}\,dz \geqslant 1$,

où c est une courbe fermée à tangente variant d'une manière continue; c ist située dans le domaine D, étant non homologue à zéro dans ce domaine.

Dans ces conditions on a pour chaque fonction de la famille

$$\pi(R^2-1) \leqslant \iint_D |f'(z)|^2\,d\sigma.$$

Le signe d'égalité a lieu dans le cas et dans ce cas seulement où la fonction $w=f(z)$ réalise la représentation conforme du domaine D sur l'anneau $1<|w|<R$.

Institut Pédagogique.
Koutaïssi.

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LITTÉRATURE CITÉE

[1] H. Grötzsch, Leipzig. Ber., VII (1928).

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MATHÉMATIQUES

SUR LE PROBLÈME DE CAUCHY POUR LES ÉQUATIONS QUASI-LINÉAIRES HYPERBOLIQUES

Par S. SOBOLEFF, membre correspondant de l'Académie

Il est connu que pour affirmer l'existence d'une solution continue et dérivable de l'équation quasi-linéaire hyperbolique:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}\left(u, \frac{\partial u}{\partial x_i}, \frac{\partial u}{\partial t}, x_1, \ldots x_n, t\right)\frac{\partial^2 u}{\partial x_i \partial x_j} - \frac{\partial^2 u}{\partial t^2} = F\left(u, \frac{\partial u}{\partial x_i}, \frac{\partial u}{\partial t}, x_1, \ldots x_n, t\right), \tag{1}$$

où

$$\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}p_i p_j > C\sum_{i=1}^{n} p_i^2, \tag{2}$$

satisfaisant aux conditions

$$u\Big|_{t=0} = u_0; \quad \frac{\partial u}{\partial t}\Big|_{t=0} = u_1, \tag{3}$$

il faut admettre l'existence d'un certain nombre de dérivées continues des fonctions u_0 et u_1. Dans toutes les recherches parues ce nombre est plus grand que le nombre correspondant des dérivées dans le cas linéaire. Un résultat assez général appartient à F. Frankl (1). Il a été obtenu au moyen d'évaluations données par l'auteur (2), (3) pour les familles de fonctions ayant les dérivées à carré sommable. Dans la Note précédente des C. R. Acad. Sci. URSS (4) nous avons indiqué quelques inégalités nouvelles qui permettent d'approfondir ce résultat. L'application des évaluations nouvelles nous permet de préciser les inégalités connues de Friedrichs et Levi et par suite de ne pas exiger dans le cas quasi-linéaire un nombre plus grand de dérivées que dans le cas linéaire.

Considérons d'abord l'équation linéaire:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} - \frac{\partial^2 u}{\partial t^2} = F \tag{4}$$

dans laquelle le second membre F et les coefficients A_{ij} dépendent seulement de $x_1, \ldots x_n$, t et ne dépendent pas de u et de ses dérivées.

Supposons que les coefficients A_{ij} et le terme F ont des dérivées jusqu'à l'ordre $\left[\frac{n}{2}\right]+2$ inclusivement dans le sens indiqué dans la Note précédente. Considérons un domaine Ω de notre espace, borné par le plan $t=0$, le plan $t=T$ et par une surface S' que nous supposons avoir partout ou bien l'orientation de l'espace ou bien l'orientation caractéristique; supposons d'ailleurs que pour cette surface l'angle entre la normale extérieure à S' et l'axe t est aigu. Soit D_t le domaine borné qui est l'intersection du plan $t=\text{const}$ avec Ω.

Supposons maintenant que les dérivées de A_{ij} et de F satisfont aux inégalités intégrales:

$$\left(\int_{D_t}\cdots\int\left[\frac{\partial^\rho A_{ij}}{\partial t^{\alpha_0}\dots\partial x_n^{\alpha_n}}\right]^2 dx_1\dots dx_n\right)^{\frac{1}{2}}\leqslant A(t)<A,$$
$$\left(\int_{D_t}\cdots\int\left[\frac{\partial^\rho F}{\partial t^{\alpha_0}\dots\partial x_n^{\alpha_n}}\right]^2 dx_1\dots dx_n\right)^{\frac{1}{2}}\leqslant F(t)<F, \tag{5}$$
$$\rho=0,1,\dots\left[\frac{n}{2}\right]+2,$$

$A(t)$ et $F(t)$ étant bornés.

Soit d'ailleurs les fonctions u_0 et u_1 telles que

$$\int_{D_0}\cdots\int\left[\frac{\partial^\rho u_0}{\partial x_1^{\alpha_1}\dots\partial x_n^{\alpha_n}}\right]^2 dx_1\dots dx_n\leqslant U,$$
$$\int_{D_0}\cdots\int\left[\frac{\partial^{\rho-1} u_1}{\partial x_1^{\alpha_1}\dots\partial x_n^{\alpha_n}}\right]^2 dx_1\dots dx_n\leqslant U, \tag{6}$$
$$\rho=0,1,\dots\left[\frac{n}{2}\right]+3.$$

On peut démontrer le lemme suivant:

Lemme I (lemme généralisé de Friedrichs et Levi).

Soit

$$K_\rho(t)=\int_{D_t}\cdots\int\sum\left[\frac{\partial^\rho u}{\partial t^{\alpha_0}\dots\partial x_n^{\alpha_n}}\right]^2 dx_1\dots dx_n. \tag{7}$$

Les fonctions $K_\rho(t)$ satisfont au système d'inégalités suivantes:

$$K_0(t)\leqslant K_0(0)+2\int_0^t K_0(t_1)^{\frac{1}{2}}K_1(t_1)^{\frac{1}{2}}dt_1,$$
$$K_{\rho+1}(t)\leqslant K_{\rho+1}(0)+N\int_0^t K_{\rho+1}(t_1)^{\frac{1}{2}}\left\{F(t_1)+A(t_1)\sum_{s=1}^{\rho+1}K_s(t_1)^{\frac{1}{2}}\right\}dt_1, \tag{8}$$
$$\rho=0,1,\dots\left[\frac{n}{2}\right]+2,$$

où la constante N dépend seulement de la forme de Ω et du nombre A et ne dépend ni de l'aspect des coefficients A_{ij}, F ni de la solution u.

La démonstration de la première inégalité (8) ne présente pas de difficultés. Pour démontrer la seconde supposons d'abord que les dérivées de u sont continues. Suivant Friedrichs et Levi considérons l'intégrale:

$$J_v(t) = \overbrace{\int\cdots\int}^{n}_{S_t}\left\{\left[\sum_{i=1}^{n}\sum_{j=1}^{n}A_{ij}\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial x_j}+\left(\frac{\partial v}{\partial t}\right)^2\right]\cos nt - \right.$$

$$\left. -2\sum_{i=1}^{n}\sum_{j=1}^{n}A_{ij}\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial t}\cos nx_j\right\} dS \equiv \overbrace{\int\cdots\int}^{n+1}_{\Omega_t}\left\{-2\left[\sum_{i=1}^{n}\sum_{j=1}^{n}A_{ij}\frac{\partial^2 v}{\partial x_i\partial x_j}-\frac{\partial^2 v}{\partial t^2}\right]\frac{\partial v}{\partial t}-\right.$$

$$\left.-\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial A_{ij}}{\partial t}\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial x_j}-2\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial A_{ij}}{\partial x_j}\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial t}\right\} dx_1\ldots dx_n\,dt. \qquad (9)$$

Ici nous désignons par Ω_t la partie du domaine $\Omega(x_1,\ldots x_n,t')$, où $t'\leqslant t$, et par S_t la surface de Ω_t.

Si nous remplaçons dans (9) la fonction v par toutes les dérivées de u d'ordre ρ et faisons l'addition de tous les résultats, nous aurons en évaluant la somme des intégrales étendues à S_t:

$$\sum_{\rho}J_v(t)\geqslant N_1K_{\rho+1}(t)-N_2K_{\rho+1}(0), \qquad (10)$$

où $N_1\geqslant\alpha>0$, α ne dépend ni d'A ni de D_t.

D'autre part, nous pouvons évaluer la somme $\sum_{\rho}J_v(t)$ de la manière suivante

$$\sum_{\rho}J_v(t)\leqslant N_3\overbrace{\int\cdots\int}^{n+1}_{\Omega_t}\left\{\sum\left|\frac{\partial^s A_{ij}}{\partial t^{\alpha_0}\ldots\partial x_n^{\alpha_n}}\right|\left|\frac{\partial^{\rho+1}u}{\partial t^{\beta_0}\ldots\partial x_n^{\alpha_n}}\right|\left|\frac{\partial^{\rho-s+2}u}{\partial t^{\gamma_0}\ldots\partial x_n^{\gamma_n}}\right|+\right.$$

$$\left.+\sum\left|\frac{\partial^{\rho+1}u}{\partial t^{\alpha_0}\ldots\partial x_n^{\alpha_n}}\right|\left|\frac{\partial^{\rho}F}{\partial t^{\beta_0}\ldots\partial x_n^{\beta_n}}\right|\right\} dx_1\ldots dx_n\,dt' =$$

$$=N_3\int_0^t\left(\sum R_{s,\rho}(t')+\sum R_{\rho}(t')\right)dt', \qquad (11)$$

où N_3 est une constante ne dépendant que de ρ et la somme est étendue à toutes les dérivées de l'ordre ρ, $s<\rho$, et $\rho-s+2$.

Nous aurons alors en tenant compte de l'inégalité de Hölder:

$$R_{s,\rho}(t')\leqslant$$

$$\leqslant\left[\int_{D_t}\cdots\int\left|\frac{\partial^s A_{ij}}{\partial t^{\alpha_0}\ldots\partial x_n^{\alpha_n}}\right|^{\frac{4n}{4s-5}}dx_1\ldots dx_n\right]^{\frac{4s-5}{4n}}\left[\int_{D_t}\cdots\int\left[\frac{\partial^{\rho+1}u}{\partial t^{\beta_0}\ldots\partial x_n^{\beta_n}}\right]^2 dx_1\ldots dx_n\right]^{\frac{1}{2}}\times$$

$$\times\left[\int_{D_t}\cdots\int\left|\frac{\partial^{\rho-s+2}u}{\partial t^{\gamma_0}\ldots\partial x^{\gamma_n}}\right|^{\frac{4n}{2n-4s+5}}dx_1\ldots dx_n\right]^{\frac{2n-4s+5}{4n}}. \qquad (12)$$

Au moyen des évaluations de la Note précédente (4) nous aurons:

$$\left.\begin{aligned} R_{s,\rho}(t') &\leqslant A(t')\left\{N_4 K_{\rho+1}(t') + N_5 K_{\rho+1}(t')^{\frac{1}{2}} K_{\rho-s+2}(t')^{\frac{1}{2}}\right\} \\ \text{et d'une façon analogue:} \\ R_\rho(t') &\leqslant N_6 F(t')\, K_{\rho+1}(t')^{\frac{1}{2}} \end{aligned}\right\} \quad (13)$$

d'où résulte le lemme I. Dans le cas, où u, A_{ij} et F n'ont pas de dérivées continues, on obtient le résultat en utilisant la méthode des fonctions moyennes (5). Du lemme I on peut déduire le théorème suivant:

Théorème I. *Soit u une solution de l'équation* (4) *satisfaisant aux conditions* (3). *Alors toutes les dérivées de u jusqu'à l'ordre* $\left[\frac{n}{2}\right]+3$ *y compris sont à carré sommable dans chaque domaine D_t pour $t < T'$. L'évaluation des intégrales de leurs dérivées dépend seulement des nombres A, F et U et de la forme du domaine Ω, mais ne dépend pas de u.*

Pour la démonstration il suffit de remarquer que

$$K_\rho(t')^{\frac{1}{2}} \leqslant \frac{1}{2}[1+K_\rho(t')]$$

et de se servir de la méthode des évaluations successives.

Du théorème I on peut déduire le théorème suivant.

Théorème II. *L'équation* (4) *admet une solution aux dérivées continues du 1-er et du 2-e ordre satisfaisant aux conditions* (3).

La démonstration se fait comme d'habitude.

Passons aux équations quasi-linéaires. Les inégalités (8) permettent de démontrer l'existence de la solution de (1) au moyen de méthodes diverses (6). Nous nous servons d'une modification de la méthode due à I. Petrowski (7). Admettons que les coefficients A_{ij} et la fonction F possèdent la propriété T) (5).

Alors le lemme suivant a lieu.

Lemme II. *On peut construire une fonction $v(x_1, \dots x_n, t)$ qui possède des dérivées jusqu'à l'ordre* $\left[\frac{n}{2}\right]+3$ *à carré sommable dans chaque D_t et telle que la différence*

$$w = u - v$$

satisfait à une équation quasi-linéaire du même type que (1) *et les dérivées de w jusqu'à l'ordre* $\left[\frac{n}{2}\right]+3$ *s'annulent pour $t=0$.*

Nous omettons ici la démonstration fondée sur la théorie des fonctions moyennes.

En se servant du lemme II ont peut réduire à zéro les valeurs initiales pour toutes les dérivées jusqu'à l'ordre $\left[\frac{n}{2}\right]+3$.

Remplaçons maintenant l'équation (1) par l'équation fonctionnelle:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}\Big(u(x_1, \dots x_n, t-h), \frac{\partial u}{\partial x_i}(x_1, \dots x_n, t-h), \frac{\partial u}{\partial t}(x_1, \dots x_n, t-h), x_1, \dots x_n, t\Big)\frac{\partial^2 u}{\partial x_i \partial x_j} - \frac{\partial^2 u}{\partial t^2} =$$

$$= F\Big(u(x_1, \dots x_n, t-h), \frac{\partial u}{\partial x_i}(x_1, \dots x_n, t-h), \frac{\partial u}{\partial t}(x_1, \dots x_n, t-h), x_1, \dots x_n, t\Big). \quad (14)$$

Quand $h=0$ nous obtenons l'équation (1).

Alors pour l'équation (14) l'évaluation suivante a lieu:

$$A(t) \leqslant K\left[1+\sum_{\rho=0}^{\left[\frac{n}{2}\right]+3} K_\rho (t-h)^{\frac{1}{2}}\right];\ F(t) \leqslant K\left[1+\sum_{\rho=0}^{\left[\frac{n}{2}\right]+3} K_\rho (t-h)^{\frac{1}{2}}\right]. \quad (15)$$

Cette inégalité découle des considérations de la Note précédente (4). Nous obtenons encore

Lemme III. *Les solutions de* (14) *qui s'annulent avec leurs dérivées pour* $t=0$ *admettent des dérivées bornées et uniformément continues des deux premiers ordres.*

Pour la démonstration on se sert de la méthode des évaluations successives qu'on applique à (9) et (15).

Les équations (14) sont résolubles pour chaque h grâce au choix des conditions initiales. En se basant sur le théorème connu d'Arzèla nous obtenons le théorème suivant.

Théorème III. *L'équation* (1) *admet une solution satisfaisant à* (3) *et possédant des dérivées continues des deux premiers ordres.*

Pour établir l'unicité de la solution il suffit de prouver que si dans un intervalle quelconque $0 \leqslant t \leqslant \delta$ l'équation est satisfaite par $u \equiv 0$, alors dans cet intervalle il ne peut exister d'autre solution.

Dans ce cas la deuxième des formules (15) peut être remplacée par:

$$F(t) \leqslant K\left[\sum_{\rho=0}^{\left[\frac{n}{2}\right]+3} K_\rho (t-h)^{\frac{1}{2}}\right].$$

L'application de la méthode des évaluations successives nous donne un résultat définitif. Nous aurons donc le théorème:

Théorème IV. *La solution de l'équation* (1) *satisfaisant aux conditions* (3) *est unique.*

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MATHÉMATIQUES

SYSTÈMES CONJUGUÉS PERMANENTS ET SYSTÈMES ORTHOGONAUX PERMANENTS DE SURFACES RÉGLÉES DES CONGRUENCES RECTILIGNES

Par S. ROSSINSKI

(Présenté par S. A. Tchapliguine, de l'Académie, le 9. V. 1938)

§ 1. Il est connu que les développables d'une congruence, les surfaces réglées principales, les surfaces distributives, les surfaces caractéristiques et, enfin, les surfaces réglées dont la représentation sphérique se compose de lignes de longueur nulle, forment le système complet de surfaces réglées de la congruence donnée [théorème de K. Ogura (1)].

La théorie de la déformation des congruences avec conservation de l'un quelconque des cinq systèmes indiqués de surfaces réglées n'est pas encore construite complètement. Pour le cas de la déformation des congruences avec développables persistantes on connaît les résultats définitifs de S. Finikoff (2). La déformation des congruences avec conservation des surfaces reglées principales (3) ainsi qu'avec conservation des surfaces distributives (4) a été considérée par l'auteur de la présente Note probablement dans tous les cas qui admettent des solutions non triviales.

Dans chacune de ces déformations il s'agit de systèmes de surfaces réglées vérifiant deux des trois propriétés suivantes: l'orthogonalité (surfaces principales et surfaces distributives), la propriété d'être isoclines (surfaces principales et surfaces développables), la propriété d'être conjuguées (surfaces distributives et surfaces développables—pour ces dernières la propriété d'être autoconjuguées) (5).

Par suite, il est naturel de poser un problème plus général concernant la déformation des congruences avec conservation d'un système de surfaces réglées vérifiant l'une quelconque des trois propriétés en question. La théorie des déformations de cette nature nous permettra de considérer d'un autre point de vue les théorèmes sur la déformation avec conservation de l'un quelconque des cinq systèmes de surfaces qui se trouvent dans le système complet.

D'ailleurs cette théorie prouve l'existence de systèmes permanents de surfaces réglées, qui ne font pas partie du système complet. En tenant compte de ce que dans la théorie de la déformation des congruences avec conservation de leurs surfaces développables, de leurs surfaces réglées principales et de leurs surfaces distributives on ne trouve des résultats dignes d'intérêt que dans le cas d'une déformation à base principale (à réseau conjugué persistant), nous nous bornons dans la présente recherche aux questions concernant la déformation à base principale.

Supposons que les rayons d'une congruence C sont invariablement liés aux plans tangents correspondants d'une surface quelconque S douée d'une base principale.

Dans la présente communication nous nous proposons d'indiquer les résultats concernant deux des trois problèmes suivants.

Supposons qu'au réseau de la base principale d'une surface S correspond un système conjugué (orthogonal ou isocline) de surfaces réglées de la congruence C conservant la propriété d'être conjugué (orthogonal ou isocline) au cours de la déformation de la surface S à base principale.

On se propose de déterminer la nature de la surface correspondante S, de la congruence C ainsi que du système conjugué (orthogonal ou isocline) permanent de surfaces réglées.

La solution du troisième problème sera l'objet d'une communication prochaine.

§ 2. Système conjugué permanent

1. Le cas d'un rayon de la congruence C orthogonal au plan tangent correspondant de la surface S. La surface S est une surface arbitraire douée d'une base principale.

Le pied du rayon sur le plan tangent est déterminé par les équations:

$$E\left[1+\xi_u+\xi\left\{\begin{matrix}11\\1\end{matrix}\right\}+\eta\left\{\begin{matrix}12\\1\end{matrix}\right\}\right]+F\left[\eta_u+\xi\left\{\begin{matrix}11\\2\end{matrix}\right\}+\eta\left\{\begin{matrix}12\\2\end{matrix}\right\}\right]=0,$$

$$G\left[1+\eta_v+\eta\left\{\begin{matrix}22\\2\end{matrix}\right\}+\xi\left\{\begin{matrix}12\\2\end{matrix}\right\}\right]+F\left[\xi_v+\eta\left\{\begin{matrix}22\\1\end{matrix}\right\}+\xi\left\{\begin{matrix}12\\1\end{matrix}\right\}\right]=0,$$

E, F, G étant les coefficients de la forme $d\bar{\rho}^2$ attachée à la surface S définie par le vecteur $\bar{\rho}(u, v)$; les fonctions ξ et η caractérisent le pied cherché en se trouvant dans l'expression du vecteur $\bar{\rho}_1(u, v)$ de la surface de départ de la congruence C:

$$\bar{\rho}_1=\bar{\rho}+\xi\bar{\rho}_u+\eta\bar{\rho}_v.$$

Cette surface de départ coïncide avec la surface moyenne de la congruence C et, par conséquent, la surface S coïncide avec l'enveloppée moyenne de C.

Le système conjugué permanent de surfaces réglées n'entre pas dans le système complet.

La congruence trouvée coïncide aves celle qui a été indiquée par l'auteur dans le problème de la déformation d'une congruence à base principale avec conservation de la surface enveloppée moyenne (6).

2. Le cas d'un rayon de la congruence C se trouvant dans le plan tangent correspondant de la surface S et d'ailleurs étant parallèle à la tangente à une courbe de la base principale.

La surface S est une surface arbitraire douée d'une base principale comprenant une famille de géodésiques.

La congruence C est formée des droites, parallèles aux tangentes des géodésiques $u=$const de la base principale et découpant sur les tangentes aux courbes $v=$const de la base principale les segments:

$$\xi=e^{\int\frac{F}{E}\left\{\begin{matrix}11\\2\end{matrix}\right\}du}\left[C(v)-\int\sqrt{E}\,e^{-\int\frac{F}{E}\left\{\begin{matrix}11\\2\end{matrix}\right\}du}\,du\right],$$

$C(v)$ étant une fonction arbitraire de v et E, F, G ayant la signification précédemment indiquée.

Le système conjugué permanent de surfaces réglées coïncide avec le système des surfaces distributives de la congruence C. J'ai déjà trouvé cette congruence C dans le problème de la déformation d'une congruence avec conservation des surfaces distributives (4).

3. Le cas d'un rayon de la congruence C découpant sur les tangentes aux courbes correspondantes de la base principale des segments ξ et η de longueurs finies et différentes de zéro.

La surface S est une surface arbitraire douée d'une base principale. Les segments ξ et η sont déterminés par les équations:

$$\eta\xi_u+\eta\sqrt{E}+\xi(\xi-\eta\cos\omega)\left\{\begin{matrix}11\\2\end{matrix}\right\}\sqrt{\frac{G}{E}}=0,$$

$$\xi\eta_v+\xi\sqrt{G}+\eta(\eta-\xi\cos\omega)\left\{\begin{matrix}22\\1\end{matrix}\right\}\sqrt{\frac{E}{G}}=0,$$

où $\cos\omega=\frac{F}{\sqrt{EG}}$. Dans le cas d'une surface de Voss on a, par suite, $\xi=-s_v+c_1(v)$, $\eta=-s_u+c_2(u)$, s_v et s_u étant les arcs des géodésiques v=const et u=const et c_1 et c_2 des fonctions arbitraires des arguments indiqués.

Le système conjugué permanent de surfaces réglées se trouve hors du système complet.

La congruence C n'est ni une congruence de normales, ni une congruence isotrope.

4. Le cas du rayon de la congruence C se trouvant dans le plan tangent correspondant de la surface S et d'ailleurs passant par le point de contact.

C'est un cas de dégénération. La surface S se réduit à une surface développable et C à une congruence cylindrique.

5. Le cas d'un rayon ayant une position arbitraire par rapport au plan tangent correspondant de la surface S.

Le problème se réduit dans ce cas à six équations par rapport aux cinq fonctions qui déterminent la position du rayon. C'est pourquoi sans une étude approfondie de ce système des équations, d'ailleurs assez compliqué, il n'est même pas permis d'affirmer qu'une déformation correspondante existe. Ainsi, dans ce cas la question reste ouverte.

§ 3. Système orthogonal permanent

1. Le cas d'un rayon de la congruence C orthogonal au plan tangent correspondant de la surface S. La surface S est une surface moulure de Monge; sa base principale est formée de lignes de courbure.

Le pied du rayon sur le plan tangent correspondant a une position arbitraire.

Le système orthogonal permanent de surfaces réglées ne fait pas partie du système complet.

2. Le cas d'un rayon de la congruence C qui se trouve dans le plan tangent correspondant de la surface S et d'ailleurs passe par le point de contact. La surface S est une surface arbitraire douée d'une base principale.

L'angle φ du rayon de la congruence C avec la tangente à la courbe v=const de la base principale sera défini par l'équation:

$$\left[\varphi_v+\left\{\begin{matrix}12\\2\end{matrix}\right\}\sqrt{\frac{G}{E}}\sin\omega\right]\left[\omega_u-\varphi_u+\left\{\begin{matrix}12\\1\end{matrix}\right\}\sqrt{\frac{E}{G}}\sin\omega\right]=$$
$$=K\sqrt{EG}\sin\varphi\sin(\omega-\varphi),$$

K étant la courbure de Gauss de la surface S et les autres quantités ayant les valeurs précédemment indiquées.

Le système orthogonal permanent de surfaces réglées n'entre pas dans le système complet.

3. Le cas d'un rayon de la congruence C coïncidant avec la tangente à une courbe d'une famille de la base principale (le cas particulier du précédent). La surface S est une surface arbitraire douée d'une base principale comprenant une famille de géodésiques.

La congruence C — de normales — est formée des tangentes à cette famille de géodésiques.

Le système orthogonal permanent de surfaces réglées coïncide avec le système des développables (réglées principales) de la congruence.

4. Le cas d'un rayon de la congruence C découpant sur les tangentes aux courbes correspondantes de la base principale des segments ξ et η de longueurs finies et différentes de zéro. La surface S est une surface arbitraire douée d'une base principale.

Les segments ξ et η sont liés par une seule équation:

$$PQ+(\xi^2+\eta^2-2\xi\eta\cos\omega)\,\xi\eta K\sqrt{EG}=0,$$

où l'on a

$$P=\xi\eta_u-\eta\xi_u+\left\{\begin{matrix}12\\1\end{matrix}\right\}\sqrt{\frac{E}{G}}\,\eta(\eta-\xi\cos\omega)+\left\{\begin{matrix}11\\2\end{matrix}\right\}\sqrt{\frac{G}{E}}\,\xi(\eta\cos\omega-\xi),$$

Q se déduisant de P à l'aide des substitutions (u, v), (ξ, η).

Le système orthogonal permanent de surfaces réglées ne fait pas partie du système complet.

5. Le cas d'un rayon de la congruence C qui se trouve dans le plan tangent et d'ailleurs est parallèle à la tangente à une courbe de la base principale (un cas particulier du précédent à condition que l'un des segments ξ, η soit infini).

La surface S est une surface arbitraire douée d'une base principale comprenant une famille de géodésiques.

La congruence C est formée des rayons parallèles aux tangentes des géodésiques de la base principale et découpant sur les tangentes aux courbes de la seconde famille de la base principale des segments d'une longueur arbitraire.

Le système orthogonal permanent de surfaces réglées de la congruence C n'entre pas dans le système complet.

6. Le cas d'un rayon de la congruence C possédant une position arbitraire par rapport au plan tangent correspondant de la surface S.

Dans ce cas les trois fonctions définissant la direction du rayon vérifient quatre équations d'ailleurs assez compliquées. Par suite, il n'est pas permis, sans une étude approfondie de ce système, d'affirmer qu'une déformation correspondante existe.

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MATHÉMATIQUES

SYSTÈMES ISOCLINES PERMANENTS DE SURFACES RÉGLÉES DES CONGRUENCES RECTILIGNES

Par S. ROSSINSKI

(Présenté par S. A. Tchaplyguine, de l'Académie, le 9. V. 1938)

§ 1. Dans la présente Note nous donnons la solution du troisième problème posé dans notre communication précédente (1).

1. Le cas d'un rayon de la congruence C orthogonal au plan tangent correspondant de la surface S.

La surface S est une surface arbitraire douée d'une base principale. Le pied du rayon sur le plan tangent est défini par les équations:

$$F\left[1+\xi_u+\xi\left\{\begin{matrix}11\\1\end{matrix}\right\}+\eta\left\{\begin{matrix}12\\1\end{matrix}\right\}\right]+G\left[\eta_u+\xi\left\{\begin{matrix}11\\2\end{matrix}\right\}+\eta\left\{\begin{matrix}12\\2\end{matrix}\right\}\right]=0,$$

$$F\left[1+\eta_v+\eta\left\{\begin{matrix}22\\2\end{matrix}\right\}+\xi\left\{\begin{matrix}12\\2\end{matrix}\right\}\right]+E\left[\xi_v+\eta\left\{\begin{matrix}22\\1\end{matrix}\right\}+\xi\left\{\begin{matrix}12\\1\end{matrix}\right\}\right]=0,$$

les quantités E, F, G, ξ, η ayant le même sens que dans la note citée (cf. § 2, n° 1).

Le système isocline permanent est formé des développables de la congruence C.

La congruence trouvée est identique à celle qui a été indiquée par S. Finikoff dans le problème de la déformation d'une congruence avec développables persistantes (2).

Outre cette solution réelle on peut signaler aussi quelques solutions imaginaires. Ce sont précisément les cas où la surface S est douée d'une base principale comprenant l'une ou les deux familles de lignes de longueur nulle (minima). Dans le premier de ces cas la congruence imaginaire C se réduit à ∞^1 cylindres; dans le second elle est douée, tout en restant imaginaire, d'un système isocline permanent formé des surfaces réglées dont la représentation sphérique est constituée des lignes de longueur nulle.

2. Le cas d'un rayon de la congruence C qui se trouve dans le plan tangent correspondant de la surface S et d'ailleurs est parallèle à la tangente à une courbe de la base principale.

La surface S est une surface arbitraire douée d'une base principale comprenant une famille de géodésiques.

La congruence C est formée par les droites parallèles aux tangentes géodésiques $u = \text{const}$ de la base principale et découpant sur les tangentes aux courbes $v = \text{const}$ de la base principale les segments

$$\xi = Ue^{\int \left[\frac{F}{E} \left\{ \begin{matrix} 12 \\ 2 \end{matrix} \right\} - \frac{KG}{\left\{ \begin{matrix} 12 \\ 1 \end{matrix} \right\}} \right] dv},$$

U étant une fonction arbitraire par rapport à u, K est la courbure de Gauss de la surface S et E, F, G sont les coefficients de son élément linéaire.

Le système isocline permanent coïncide avec le système des surfaces réglées principales de la congruence C.

La solution en question est un cas particulier de celle qui a été trouvée par l'auteur pour le problème de la déformation d'une congruence avec conservation des surfaces réglées principales dans le cas d'une position analogue du rayon (3).

3. Le cas d'un rayon de la congruence C coïncidant avec la tangente d'une courbe d'une famille de la base principale. La surface S est une surface arbitraire douée d'une base principale.

Le système isocline permanent est formé des développables de la congruence C.

Cette congruence C a été trouvée par S. Finikoff dans sa Thèse citée (cf. ch. II, p. 49, en russe).

§ 2. Le cas d'un rayon de la congruence C découpant sur les tangentes aux courbes correspondantes de la base principale des segments ξ et η de longueurs finies et différentes de zéro.

Ce cas fournit trois solutions essentiellement différentes.

1. La surface S est une surface arbitraire douée d'une base principale. Les segments ξ et η se trouvent des équations:

$$q \equiv \xi\eta_u + \eta\sqrt{E} + \eta(\eta - \xi\cos\omega)\left\{ \begin{matrix} 12 \\ 1 \end{matrix} \right\}\sqrt{\frac{E}{G}} = 0,$$

$$n \equiv \eta\xi_v + \xi\sqrt{G} + \xi(\xi - \eta\cos\omega)\left\{ \begin{matrix} 12 \\ 2 \end{matrix} \right\}\sqrt{\frac{G}{E}} = 0,$$

ω étant l'angle des lignes coordonnées (courbes de la base principale).

Le système isocline permanent est formé des développables de la congruence C trouvée par S. Finikoff (l. c., pp. 50—51).

2. La surface S est une surface arbitraire douée d'une base principale. Les segments ξ et η sont définis par les équations:

$$pq - mn = 0,$$

$$pm + qn - 2mn + \xi\eta(\xi^2 + \eta^2 - 2\xi\eta\cos\omega)K\sqrt{EG} = 0,$$

où

$$m \equiv \eta\xi_u + \eta\sqrt{E} + \xi(\xi - \eta\cos\omega)\left\{ \begin{matrix} 11 \\ 2 \end{matrix} \right\}\sqrt{\frac{G}{E}},$$

p se déduit de m à l'aide des substitutions (u, v), (ξ, η) et q et n ont les mêmes valeurs que dans le cas précédent.

Le système isocline permanent est formé des surfaces réglées principales de la congruence C. Cette congruence C a été trouvée par l'auteur pour le problème de la déformation d'une congruence avec conservation des surfaces réglées principales (3).

3. La surface S est une surface arbitraire douée d'une base principale.

On calcule les segments ξ et η en résolvant les équations:

$$pq + mn - 2qn = 0,$$
$$qn - pm + \xi\eta(\xi^2 + \eta^2 - 2\xi\eta\cos\omega) K\sqrt{EG} = 0,$$

m, n, p, q ayant des valeurs précédemment indiquées.

Le système isocline permanent de surfaces réglées de la congruence C ne fait pas partie du système complet.

§ 3. Quant à la possibilité de la déformation considérée dans le cas d'un rayon, se trouvant dans le plan tangent correspondant de la surface S, passant par le point de contact et d'ailleurs différent de la tangente à une courbe de la base principale, ainsi que dans le cas d'un rayon, possédant une position arbitraire par rapport au plan tangent de la surface S, on trouve que dans le premier cas le problème se réduit à deux ou trois équations par rapport à une fonction et dans le second cas à huit équations par rapport aux cinq fonctions définissant la position du rayon et, par suite, sans une étude approfondie des systèmes correspondants, d'ailleurs très compliqués, on ne peut même pas affirmer qu'une déformation correspondante existe.

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MATHÉMATIQUES

SUR LA RÉDUCTION DES ÉQUATIONS INTÉGRALES SINGULIÈRES À DES ÉQUATIONS ÉQUIVALENTES DE FREDHOLM

Par S. G. MICHLIN

(Présenté par I. M. Vinogradow, de l'Académie, le 15. V. 1938)

Nous avons démontré (1) que chaque opérateur singulier

$$Tw = a(M)W(M) + \int_E W(M_1)\frac{f(M;\Theta_1, \dots \Theta_{m-2}, \varphi)}{R^m} d\tau_1 * \qquad (1)$$

(E désignant l'espace euclidien à m dimensions) peut être développé en une série suivant les puissances de certains opérateurs $U_1, U_2, \dots U_{m-2}, U_{m-1}$. Les symboles de ces opérateurs sont respectivement égaux à $e^{2i\Theta_1}, e^{2i\Theta_2}, \dots e^{2i\Theta_{m-2}}, e^{i\varphi}$ (1); les opérateurs eux-mêmes sont commutatifs et unitaires dans l'espace L_2:

$$T = \sum_{-\infty}^{+\infty} a_{n_1 n_2 \dots n_{m-1}}(M)\, U_1^{n_1} U_2^{n_2} \dots U_{m-1}^{n_{m-1}}. \qquad (2)$$

En remplaçant dans (2) les U_k par leurs symboles nous obtiendrons le symbole de l'opérateur T.

Lemme. *La norme de l'opérateur T ne surpasse pas le maximum du module de son symbole.*

Déterminons les opérateurs $A_1, \dots A_{m-1}$, dont chacun est associé à lui-même, par les relations

$$U_k = e^{2\pi i A_k}. \qquad (3)$$

Soit

$$T = \sum_0^\infty b_{n_1 n_2 \dots n_{m-1}}(M)\, A_1^{n_1} A_2^{n_2} \dots A_{m-1}^{n_{m-1}} = F(M; A_1, A_2, \dots A_{m-1}). \qquad (4)$$

Désignons par $E_k(\lambda_k)$ la décomposition de l'unité liée à l'opérateur A_k. Il est connu que

$$A_1^{n_1} A_2^{n_2} \dots A_{m-1}^{n_{m-1}} W = \int_0^{1(m-1)} \lambda_1^{n_1} \lambda_2^{n_2} \dots \lambda_{m-1}^{n_{m-1}} dE_0 W, \qquad (5)$$

où

$$dE_0 = dE_1(\lambda_1)\, dE_2(\lambda_2) \dots dE_{m-1}(\lambda_{m-1}).$$

* Nous conservons ici les notations de la note (1).

Dans la relation (5) la somme dont la limite est l'intégrale du second converge fortement vers le premier membre.

En multipliant (5) par les coefficients correspondants et en prenant la somme de ces termes nous obtenons:

$$TW = \int_0^{1\,(m-1)} F(M; \lambda_1, \lambda_2, \ldots \lambda_{m-1})\, dE_0 W. \tag{6}$$

Remarquons que l'ensemble des valeurs de F coïncide avec celui du symbole de l'opérateur T.

Il suit de (6)

$$\| TW \|^2 = \int_0^{1\,(m-1)} (F\, dE_0 W, TW),$$

d'où

$$\| TW \| \leqslant \max |F| \cdot \| W \|. \tag{7}$$

Théorème. *Si le symbole de l'opérateur T ne prend pas la valeur zéro, l'équation intégrale singulière*

$$TW = f(M) \tag{8}$$

est équivalente à une certaine équation de Fredholm. Dans l'équation (8) *$f(M)$ désigne la somme d'une fonction connue appartenant à L_2 et d'un opérateur totalement continu dans L_2.*

Nous avons démontré ce théorème pour $m = 2$ ([2], [3]). Cependant, notre démonstration reste valable pour m arbitraire, si le développement (2) ne contient que des puissances d'un seul opérateur. Supposons que notre théorème est vrai quand le développement (2) contient des puissances de $m-2$ opérateurs. Nous démontrerons que ce théorème est aussi vrai quand le développement (2) contient des puissances de $m-1$ opérateurs. Développons T en une série suivant les puissances de U_1:

$$T = \sum_{-\infty}^{+\infty} \rho_n (M; U_2, \ldots U_{m-1})\, U_1^n. \tag{9}$$

Posons

$$T_n = \sum_{-n}^{+n} \rho_k U_1^k, \tag{10}$$

$$T = T_n + R_n. \tag{11}$$

Choisissons n assez grand pour que le symbole T_n ne prenne pas la valeur zéro. Écrivons (8) sous la forme

$$T_n W = f - R_n W = f_1. \tag{12}$$

Considérons pour le moment f_1 comme une fonction connue. Décomposons T_n en facteurs:

$$T_n = U_1^{-n} \prod_{k=1}^{2n} [\mu_k (M; U_2, \ldots U_{m-1})\, U_1 - \nu_k (M; U_2, \ldots U_{m-1})]. \tag{13}$$

L'égalité (13) est vraie à un certain opérateur totalement continu près.

Nous écrivons l'équation (12) sous la forme:

$$\prod_{k=1}^{2n} (\mu_k U_1 - \nu_k) W = U_1^n f_2 = f_3.$$

Ici f_2 se distingue de f_1 par un opérateur totalement continu.

Posons

$$\prod_{k=1}^{2n-1} (\mu_k U_1 - \nu_k) W = V_1, \tag{14}$$

or

$$\mu_{2n} U_1 V_1 - \nu_{2n} V_1 = f_3. \tag{15}$$

Les symboles des facteurs du produit (13) ne prennent pas la valeur zéro. Conservons pour les symboles les mêmes notations que pour leurs opérateurs. Il est nécessaire qu'on ait constamment ou bien $|\mu_{2n}| > |\nu_{2n}|$, ou brien $|\nu_{2n}| > |\mu_{2n}|$.

Soit, par exemple, $|\nu_{2n}| > |\mu_{2n}|$, alors le symbole ν_{2n} ne prend pas la valeur zéro. Posons

$$\nu_{2n} V_1 = V_0. \tag{16}$$

En considérant (16) comme une équation par rapport à l'inconnue V_1, on peut la réduire à une équation équivalente de Fredholm:

$$V_1 + tV_1 = \nu_{2n}^{-1} V_0 + t' V_0^*,$$

où t et t' sont des opérateurs totalement continus. En substituant cette expression dans (15), nous obtiendrons:

$$V_0 - \mu_{2n} U_1 \nu_{2n}^{-1} V_0 = f_4 = -f_3 - \mu_{2n} U_1 (tV_1 - t' V_0). \tag{17}$$

La norme du second terme du premier membre est inférieure à l'unité. On peut résoudre l'équation (17) par la méthode des approximations successives, en considérant le second membre comme connu. Dans la solution de (17) nous remplaçons V_0 d'après (16) et V_1 d'après (14). Nous obtenons ainsi l'équation

$$\prod_{k=1}^{2n-1} (\mu_k U_1 - \nu_k) W = f_5.$$

Posons

$$\prod_{k=1}^{2n-2} (\mu_k U_1 - \nu_k) W = V_2$$

etc. Enfin nous obtiendrons l'équation

$$W + R_n T_n^{-1} W = \Phi, \tag{18}$$

où Φ est la somme d'une fonction connue et d'un opérateur totalement continu. Pour n assez grand nous avons

$$\| T_n^{-1} \| \leqslant \max \frac{1}{|T_n|} < \max \frac{2}{|T|},$$

et $\| R_n \|$ est aussi petit que l'on veut. Or, pour n assez grand, on a:

$$\| R_n T_n^{-1} \| < 1.$$

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* Ici ν_{2n}^{-1} désigne un opérateur singulier dont le symbole est égal à $\frac{1}{\nu_{2n}}$. Nous conservons le même sens pour la notation T_n^{-1} dans la formule (18).

On peut résoudre (18) par la méthode des approximations successives, et cela nous amène à une équation de Fredholm qui est équivalente à l'équation (18).

Quant aux systèmes d'équations singulières nous avons le théorème suivant:

Si le déterminant symbolique du système ne prend pas la valeur zéro, et le nombre de dimensions de l'espace est $m \geqslant 2$, le système des équations intégrales singulières est équivalent à un certain système d'équations de Fredholm.

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MATHEMATIK

ÜBER DIE SYLOWUNTERGRUPPEN DER EINFACHEN GRUPPEN

Von S. A. TCHOUNIKHIN

(Vorgelegt von I. M. Vinogradow, Mitglied der Akademie, d. 9. V. 1938)

1. In meiner Arbeit «Über einige Sätze der Gruppentheorie» (1) habe ich den folgenden Satz bewiesen:

Satz I. *Sei $\mathfrak{H}$ eine Untergruppe vom Index k der Gruppe $\mathfrak{G}$ und sei $\mathfrak{K}$ eine Untergruppe von $\mathfrak{H}$, die die Kommutatorgruppe von $\mathfrak{H}$ umfasst. Ist in $\mathfrak{H}$, aber nicht in $\mathfrak{K}$, ein Element A von zu k relativ primärer Ordnung enthalten mit der Eigenschaft, dass für jedes λ alle Elemente von $\mathfrak{H}$, die mit A^λ in $\mathfrak{G}$ konjugiert sind, dem System $\mathfrak{K}A^\lambda$ angehören, so ist die Gruppe $\mathfrak{G}$ nicht einfach.*

In der vorliegenden Arbeit wollen wir mit Hilfe des Satzes I eine Eigenschaft der Sylowuntergruppen der einfachen Gruppen feststellen. Eine andere Anwendung des Satzes I war auch von L. Weisner gegeben (2).

2. $\mathfrak{G}$ bezeichnet hier eine Gruppe der Ordnung $p^\alpha n$ (p ist eine Primzahl, n ist durch p nicht teilbar) und $\mathfrak{P}$ ist eine der Untergruppen der Ordnung p^α. Sei $\mathfrak{P}$ eine Abelsche Gruppe. Wir setzen ausserdem voraus, das $p-1$ und n teilerfremd sind. Wir beweisen zunächst die folgenden drei Hilfssätze.

Hilfssatz I. *Sei A ein Element von der Ordnung p^m der fundamentalen Basis der Gruppe $\mathfrak{P}$. Es sei ferner α durch p nicht teilbar, $m_1 < \beta, m_1 < m$. Dann ist die Gleichung*

$$A^{\alpha p^{m_1}} = X^{p^\beta} \tag{1}$$

unmöglich für jedes Element X von $\mathfrak{P}$.

Beweis. Ist X ein Element von $\mathfrak{P}$, für welches die Gleichung (1) gilt, so stellen wir dieses Element durch die Elemente der fundamentalen Basis von $\mathfrak{P}$ dar:

$$X = A^\gamma B,$$

wo B ein Element von $\mathfrak{P}$ ist, das von A bereits nicht abhängt. Also haben wir:

$$A^{\alpha p^{m_1}} = A^{\gamma p^\beta} B^{p^\beta},$$

oder

$$A^{p^{m_1}(\alpha - \gamma p^{\beta - m_1})} = B^{p^\beta}; \tag{2}$$

dabei ist, laut der Bedingungen, $\beta - m_1 > 0$. Die linke Seite von (2) ist von der Einheit verschieden, weil $m_1 < m$ und $\alpha - yp^{\beta - m_1} \equiv 0 \pmod{p}$ ist. Deshalb ist die Gleichung (2) unmöglich, wenn $B^{p^\beta} = E$. Ebenso ist diese Gleichung unmöglich, wenn $B^{p^\beta} \neq E$, weil jede von E verschiedene Potenz von A von anderen Elementen der fundamentalen Basis unabhängig ist. Damit ist der Hilfssatz I bewiesen.

Hilfssatz II. *Sei A der Ordnung p^m das einzige Element von dieser Ordnung der fundamentalen Basis der Gruppe $\mathfrak{P}$. Dann lässt sich jedes Element von $\mathfrak{P}$, das mit A in $\mathfrak{G}$ konjugiert ist, in folgender Form darstellen: $A^x B$, wo $x \equiv 1 \pmod{p}$ und B ein von A unabhängiges Element von $\mathfrak{P}$, oder E ist.*

Beweis. Es ist bekannt (3), dass die Elemente des Zentrums einer Sylowuntergruppe, die in der ganzen Gruppe konjugiert sind, bereits im Normalisator dieser Sylowuntergruppe konjugiert sind, d. h. aus der Gleichung $G^{-1}AG \subset \mathfrak{P}$, $G \subset \mathfrak{G}$ folgt $C^{-1}AC = G^{-1}AG$, $C^{-1}\mathfrak{P}C = \mathfrak{P}$. Wenn $C^{-1}AC$ von A verschieden ist, so ist C nicht in $\mathfrak{P}$ enthalten (weil $\mathfrak{P}$ in unserem Fall Abelsch ist). Stellen wir das Element $C^{-1}AC$ durch die Elemente der fundamentalen Basis dar. Es ist offenbar notwendig nur den Fall zu betrachten, wenn $C^{-1}AC$ von A verschieden ist.

Es sind die folgenden drei Fälle möglich.

1) $C^{-1}AC = A^x$. Wie ich früher gezeigt habe (4), ist jedes Element des Zentrums einer zu p gehörigen Sylowuntergruppe mit keiner seiner von ihm selbst verschiedenen Potenz konjugiert, wenn $p-1$ und die Ordnung der ganzen Gruppe teilerfremd sind. Deshalb haben wir $x \equiv 1 \pmod{p^m}$.

2) $C^{-1}AC = A^x B$, wo $B \neq E$ nur von solchen Elementen der fundamentalen Basis von $\mathfrak{P}$ abhängt, deren Ordnungen kleiner als p^m sind.

Wegen $B^{p^{m-1}} = E$ haben wir:

$$C^{-1}A^{p^{m-1}}C = A^{xp^{m-1}}.$$

Nach dem vorhergehenden Fall ist die letzte Gleichung nur dann möglich, wenn $x \equiv 1 \pmod{p}$.

3) $C^{-1}AC = A B_1 B_2$, wo $B_1 \neq E$ nur von solchen Elementen der fundamentalen Basis von $\mathfrak{P}$ abhängt, deren Ordnungen grösser als p^m sind, und B_2—von solchen, deren Ordnungen kleiner als p^m sind.

Zunächst ist es offenbar, dass die höchste Potenz eines jeden in B_1 auftretenden Elementes der fundamentalen Basis einen durch p teilbaren Exponenten hat, denn andernfalls wäre die Ordnung der rechten Seite der Gleichung $C^{-1}AC = A^x B_1 B_2$ grösser, als die Ordnung p^m der linken Seite, was unmöglich ist. Deshalb lässt sich B_1 in der Form $B_1 = D^p$ darstellen, wo D ein Element von $\mathfrak{P}$ ist. Aus $C^{-1}AC = A^x D^p B_2$ erhalten wir

$$C^{-1}A^{p^{m-1}}C = A^{xp^{m-1}}D^{p^m}, \tag{3}$$

weil $B_2^{p^{m-1}} = E$ ist. Es sei C^d die kleinste Potenz von C, die mit $A^{p^{m-1}}$ vertauschbar ist. Die Zahl d ist offenbar ein Teiler der Ordnung von C und deshalb (wie oben erwähnt ist C in $\mathfrak{P}$ nicht enthalten)

$$d \text{ und } p-1 \text{ teilerfremd sind.} \tag{4}$$

In der Reihenfolge erhalten wir aus (3)

$$C^{-2}A^{p^{m-1}}C^2 = A^{x^2p^{m-1}}D^{xp^m}C^{-1}D^{p^m}C,$$

. . . . . .

. . . . . .

$$C^{-d}A^{p^{m-1}}C^d = A^{x^dp^{m-1}}D^{x^{d-1}p^m}\underbrace{C^{-1}D^{x^{d-2}p^m}C}\ldots\underbrace{C^{-(d-1)}D^{p^m}C^{d-1}}.$$

Alle untergeklammerten Glieder sind in $\mathfrak{P}$ enthalten, weil C mit $\mathfrak{P}$ vertauschbar ist. Deshalb lässt sich die letzte Gleichung in der Form

$$C^{-d}A^{p^{m-1}}C^d = A^{x^dp^{m-1}}D_1^{p^m} \tag{5}$$

schreiben, wo D_1 auch ein Element aus $\mathfrak{P}$ ist.

Nach (5) erhalten wir

$$A^{p^{m-1}(x^d-1)} = D_2^{p^m}, \tag{6}$$

wo $D_2 = D_1^{-1}$ ebenfalls in $\mathfrak{P}$ enthalten ist.

Nach Hilfssatz I ist die Gleichung (6) nur in dem Fall möglich, wenn $x^d \equiv 1 \pmod p$ ist. Aber diese Kongruenz erfüllt sich nach (4) nur in dem Fall, wenn $x \equiv 1 \pmod p$ ist.

Damit ist der Hilfssatz II bewiesen.

Hilfssatz III. *Sei A dasselbe Element, wie in Hilfssatz* II. *Dann lässt sich jedes Element von $\mathfrak{P}$, das mit A^λ in $\mathfrak{G}$ konjugiert ist, in folgender Form darstellen: A^xB, wo $x \equiv \lambda \pmod p$ und B ein von A unabhängiges Element von $\mathfrak{P}$ oder E ist.*

Beweis. Es sei $C^{-1}A^\lambda C \subset \mathfrak{P}$, $C^{-1}\mathfrak{P}C = \mathfrak{P}$. Daraus folgt $C^{-1}A^\lambda C = -(C^{-1}AC)^\lambda$. Aber $C^{-1}AC \subset \mathfrak{P}$, weil $C^{-1}\mathfrak{P}C = \mathfrak{P}$, und nach dem Hilfssatz II ist $C^{-1}AC = A^xB$, $x \equiv 1 \pmod p$ und B ist von A unabhängig. Also haben wir $C^{-1}A^\lambda C = (A^xB)^\lambda = A^{x\lambda}B^\lambda$, wobei offenbar $x\lambda \equiv \lambda \pmod p$ und B^λ von A unabhängig (oder gleich E) ist. Damit ist Hilfssatz III bewiesen.

Satz II. *Sei $\mathfrak{G}$ eine einfache Gruppe der Ordnung $p^\alpha n$ (p ist eine Primzahl, n ist durch p nicht teilbar, $p-1$ und n sind teilerfremd). Sei $\mathfrak{P}$ eine der Untergruppen der Ordnung p^α und sei $\mathfrak{P}$ Abelsch. Dann ist die Anzahl der Elementen von derselben Ordnung der fundamentalen Basis von $\mathfrak{P}$ stets grösser als 1.*

Beweis. Sei A der Ordnung p^m ein einziges Element von dieser Ordnung der fundamentalen Basis von $\mathfrak{P}$. Sei $\mathfrak{K}$ eine solche Untergruppe von $\mathfrak{P}$, die von A^p und von allen anderen von A verschiedenen Elementen der fundamentalen Basis von $\mathfrak{P}$ erzeugt wird. Offenbar ist A in $\mathfrak{K}$ nicht enthalten.

Nach den Hilfssätzen II und III ist jedes Element von $\mathfrak{P}$, das mit A^λ in $\mathfrak{G}$ konjugiert ist, in folgender Form darstellbar: A^xB, wo $x \equiv \lambda \pmod p$ und B ein von A unabhängiges Element von $\mathfrak{P}$ oder E ist. Wir haben also $x = \lambda + tp$ und $A^xB = A^\lambda A^{tp}B$. Die Elemente A^{tp} und B sind offenbar in $\mathfrak{K}$ enthalten, und folglich gehört A^xB dem System $A^\lambda\mathfrak{K}$ an. Also erfüllt A alle Bedingungen des Satzes I, wenn man $\mathfrak{S} = \mathfrak{P}$ setzt. Also ist die Gruppe $\mathfrak{G}$ nicht einfach.

Dieser Widerspruch beweist den Satz II.

3. Als Sonderfall des Satzes II erhalten wir den folgenden Satz von L. Weisner (2):

Sei $\mathfrak{P}$ der Ordnung p^α eine Abelsche Sylowuntergruppe der Gruppe $\mathfrak{G}$, seien $p-1$ und die Ordnung von $\mathfrak{G}$ teilerfremd, und sei $\mathfrak{P}$ vom Typus $(n_1, n_2, \ldots, n_r)$. Wenn $n_1 > n_2 \geqslant \ldots \geqslant n_r$, so enthält $\mathfrak{G}$ einen Normalteiler vom Index p.

4. In ähnlicher Weise kann man auch den allgemeinen Fall, wenn $\mathfrak{P}$ nicht kommutativ ist, betrachten. Wir erhalten dann die folgenden Resultate.

Sei $\mathfrak{P}$ eine Sylowuntergruppe der Ordnung p^δ einer einfachen Gruppe $\mathfrak{G}$ der Ordnung $p^\delta n$. Es seien ferner $p-1$ und n teilerfremd. Sei $\mathfrak{L}$ der Ordnung p^η das Zentrum der Gruppe $\mathfrak{P}$ und sei $\mathfrak{L}$ vom Typus $(p^{\lambda_1}, p^{\lambda_2}, \ldots p^{\lambda_r})$, wo $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_r$. Führen wir die folgenden Bezeichnungen ein:

1) für $1 < i < r$ sei ω_i die kleinste der Zahlen $\lambda_{i-1} - \lambda_i$ und $\lambda_i - \lambda_{i+1}$;
2) für $i = 1$ sei $\omega_1 = \lambda_1 - \lambda_2$;
3) für $i = r$ sei $\omega_r = \lambda_{r-1} - \lambda_r$;
4) für $r = 1$ sei $\omega_1 = \lambda_1$.

Dann gilt

Satz III. *Für jedes i ist* $\omega_i \leqslant \delta - \eta$.

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THEORY OF ELASTICITY

TORSION OF A PRISMATIC BAR WHICH IS SIMULTANEOUSLY SUBJECTED TO TENSION

By P. M. RIZ and N. V. ZVOLINSKY

(Communicated by S. A. Tchapligin, Member of the Academy, 9.V.1938)

If we wish to remain within the scope of the linear theory of elasticity, which neglects the squares and the products of displacement derivatives, we must treat the torsion of a prism independently of tension. The state of stress under the combined action of both loads is obtained by imposing the two systems of stresses. The result is contradictory to experimental data and the present article aims to determine the effect of tension (or compression) stresses on the torsion stiffness of a prism. In order to solve this problem it was necessary to give up the linear theory of elasticity.

Let a, b and c be the co-ordinates of a material point of the body in a rectilinear system of co-ordinates before deformation and x, y, z—the co-ordinates in the same system after deformation.

Let the displacements u, v and w be given, which we may consider either as functions of the initial co-ordinates a, b, c, or as functions of the final co-ordinates x, y, z.

We now introduce the deformation components determined by the following formulae [1]:

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} - \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right], \\ \varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}. \end{aligned}\right\} \qquad (1)$$

The quantity ε_{ii} is connected with elongation e_{ii} by the following relation:

$$e_{ii} = \frac{1 - \sqrt{1 - 2\varepsilon_{ii}}}{\sqrt{1 - 2\varepsilon_{ii}}}. \qquad (2)$$

In the second approximation

$$e_{ii} = \varepsilon_{ii} + \frac{5}{2}\varepsilon_{ii}^2. \qquad (3)$$

In order to determine the stresses it is necessary to introduce some principle according to which the elongations depend on the stresses.

It may be considered as experimentally established that for finite displacements not exceeding some small value (the limit of proportionality) the principal stresses σ_i are connected with the principal elongations e_i by the following relation:

$$\sigma_i = \lambda \sum_{a=1}^{3} e_a + 2Ge_i , \tag{4}$$

wherefrom we can calculate the specific work Φ of the deformation

$$\Phi = G\left\{ \sum_{a=1}^{3} e_a^2 + \frac{\nu}{1-2\nu}\left(\sum_{a=1}^{3} e_a \right)^2 \right\}. \tag{5}$$

Using formula (3) and introducing the deformation tensor invariants:

$$I_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \; ; \; I_2 = \varepsilon_{xx}\,\varepsilon_{yy} + \varepsilon_{xx}\,\varepsilon_{zz} + \varepsilon_{yy}\,\varepsilon_{zz} - \frac{1}{4}(\varepsilon_{xy}^2 + \varepsilon_{xz}^2 + \varepsilon_{yz}^2) \; ;$$

$$I_3 = \begin{vmatrix} \varepsilon_{xx} & \frac{1}{2}\varepsilon_{xy} & \frac{1}{2}\varepsilon_{xz} \\ \frac{1}{2}\varepsilon_{xy} & \varepsilon_{yy} & \frac{1}{2}\varepsilon_{yz} \\ \frac{1}{2}\varepsilon_{xz} & \frac{1}{2}\varepsilon_{yz} & \varepsilon_{zz} \end{vmatrix},$$

let us now express the specific work of deformation in the following form:

$$\Phi = G\frac{1-\nu}{1-2\nu} I_1^2 - 2GI_2 + lI_1^3 + mI_1 I_2 + nI_3 . \tag{6}$$

According to our hypotheses as to Hook's law we have:

$$l = 3G\,\frac{1-\nu}{1-2\nu} ; \qquad m = -3G\,\frac{1-4\nu}{1-2\nu} ; \qquad n = 9G .$$

The Murnaghan formulae (2) enable us to determine the stresses provided that the coefficients in the expressions for the energy are known:

$$\left.\begin{aligned} \sigma_{xx} &= \lambda I_1 + (3l + m - \lambda) I_1^2 + mI_2 + \left\{ 2G - (m + 2\lambda + 2G) I_1 \right\} \varepsilon_{xx} + \\ &\quad + n\frac{\partial I_3}{\partial \varepsilon_{xx}} - 4G\left[\varepsilon_{xx}^2 + \frac{1}{4}(\varepsilon_{xy}^2 + \varepsilon_{xz}^2) \right], \\ \sigma_{xy} &= \left\{ G - \left(\frac{m}{2} + \lambda + G \right) I_1 \right\} \varepsilon_{xy} + 2n\frac{\partial I_3}{\partial \varepsilon_{xy}} - \\ &\quad - 2G\left[\varepsilon_{xx}\,\varepsilon_{yx} + \varepsilon_{yy}\,\varepsilon_{yx} + \frac{1}{2}\varepsilon_{xz}\,\varepsilon_{yz} \right]. \end{aligned}\right\} \tag{7}$$

The remaining stress components are determined in an analogous way.

Let us now suppose that in the case of a simultaneous action of a torque-moment applied at the end of the bar and a tension uniformly distributed upon the end section of the bar the values of u, v and w are determined by the formulae:

$$\left.\begin{aligned} u &= -\tau bc - \nu\alpha a , \\ v &= \tau ac - \nu\alpha b , \\ w &= \tau\varphi(a,b) + \alpha c , \end{aligned}\right\} \tag{8}$$

where τ is the torsion, α is a constant, and the origin is placed into the centre of gravity of one of the bases of the prism.

From the equations

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial a}\frac{\partial a}{\partial x}+\frac{\partial u}{\partial b}\frac{\partial b}{\partial x}+\frac{\partial u}{\partial c}\frac{\partial c}{\partial x}=\frac{\partial u}{\partial a}\left(1-\frac{\partial u}{\partial x}\right)-\frac{\partial u}{\partial b}\frac{\partial v}{\partial x}-\frac{\partial u}{\partial c}\frac{\partial w}{\partial x} \tag{9}$$

we determine the values: $\frac{\partial u}{\partial x}$; $\frac{\partial u}{\partial y}$; $\frac{\partial u}{\partial z}$; $\frac{\partial v}{\partial x}$ ... in terms of the derivatives $\frac{\partial u}{\partial a}$, $\frac{\partial u}{\partial b}$ ...

Introducing these values into the formula (1) we obtain the expressions for the deformation components ε_{ij} in terms of the co-ordinates of the undistorted state.

Here and in what follows all calculations are made neglecting the terms of the third and higher orders of smallness as well as terms of the second order containing the squares of τ *.

Thus we obtain

$$\left.\begin{aligned}&\varepsilon_{xx}=-\nu\alpha\,,\quad \varepsilon_{yy}=-\nu\alpha\,,\quad \varepsilon_{zz}=\alpha\,,\quad \varepsilon_{xy}=0\,,\\&\varepsilon_{xz}=\tau\left\{\varphi'_a(1-2\alpha+\nu\alpha)-b\,(1-\alpha+2\nu\alpha)\right\},\\&\varepsilon_{yz}=\tau\left\{\varphi'_b\,(1-2\alpha+\nu\alpha)+a\,(1-\alpha+2\nu\alpha)\right\};\end{aligned}\right\} \tag{10}$$

hence, according to the formulae (7),

$$\left.\begin{aligned}&\sigma_{xx}=\sigma_{yy}=\sigma_{xy}=0\,,\quad \sigma_{zz}=2G\alpha\,(1+\nu)\,,\\&\sigma_{xz}=G\tau\left(1+\frac{3}{2}\alpha+5\alpha\nu\right)\left\{\varphi'_a(1-2\alpha+\alpha\nu)-b\,(1-\alpha+2\alpha\nu)\right\},\\&\sigma_{yz}=G\tau\left(1+\frac{3}{2}\alpha+5\alpha\nu\right)\left\{\varphi'_b\,(1-2\alpha+\alpha\nu)+a\,(1-\alpha+2\alpha\nu)\right\}.\end{aligned}\right\} \tag{11}$$

It is easy to see that the conditions of equilibrium are satisfied in the case when body forces are absent, if

$$\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}=0\,,$$

which leads to the condition

$$\frac{\partial^2\varphi}{\partial a^2}+\frac{\partial^2\varphi}{\partial b^2}=0\,. \tag{12}$$

The condition of absence of stresses on the lateral surface leads to the relation

$$\frac{\partial\varphi}{\partial a}\frac{db}{ds}-\frac{\partial\varphi}{\partial b}\frac{da}{ds}=(1-\alpha-\alpha\nu)\frac{d}{ds}\left(\frac{a^2+b^2}{2}\right) \tag{13}$$

on the contour of the cross-section.

Hence we see that the function φ differs from the function $\bar{\varphi}$ corresponding to the case of pure torsion by the factor

$$[1-\alpha-\alpha\nu].$$

The torque-moment is determined by the formula

$$\left.\begin{aligned}&M=\iint\left\{x\sigma_{yz}-y\sigma_{xz}\right\}dx\,dy=\\&=\iint\left\{(a+u)\,\sigma_{yz}-(b+v)\,\sigma_{xz}\right\}\left|\frac{D(x,y)}{D(a,b)}\right|_{z=\text{const}}da\,db\,,\\&\iint\sigma_{xz}\,dx\,dy=\iint\sigma_{yz}\,dx\,dy=0\,,\end{aligned}\right\} \tag{14}$$

since the origin lies in the centre of gravity of the cross-section.

* The torsion is considered small in the classical sense.

Carrying out the calculations according to formulae (11) and (13) we obtain:

$$M = GT_0\tau\left\{1+\frac{p}{G}\,\frac{1}{1+\nu}\left[\frac{J_p}{T_0}-\frac{3}{4}+\nu\right]\right\}, \qquad (15)$$

where T_0 is the geometrical torsion stiffness of the bar not subjected to tension, J_p is the polar moment about the torque-axis and $p=2G(1+\nu)\alpha$ are the tension stresses.

Hence we obtain the main result: the geometrical torsion stiffness of a bar subjected simultaneously to the action of tension (or compression) is determined by the formula:

$$T = T_0\left\{1+\frac{p}{G(1+\nu)}\left[\frac{J_p}{T_0}-\frac{3}{4}+\nu\right]\right\}. \qquad (16)$$

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MATHEMATISCHE PHYSIK

ZUR THEORIE DER GITTER

Von V. S. IGNATOVSKIJ (W. IGNATOWSKY), correspondierendem Mitglied der Akademie

Wir betrachten wieder die Gleichung:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0, \qquad (1)$$

welche in dem Halbraume, rechts von der Ebene ZY (x positiv) gültig sein soll. Wir haben im Auge ein ebenes Problem, also Unabhängigkeit von der Z-Koordinate.

Als Integrale haben wir, analog (9) unserer Arbeit (1)

$$u(x,y,t)=-\frac{1}{\pi}\int_x^{ct} d\tau_1 \int_{-\sqrt{\tau_1^2-x^2}}^{\sqrt{\tau_1^2-x^2}} \frac{H(y-s,t-\tau)}{\sqrt{\tau_1^2-x^2-s^2}}\,ds;\ \tau_1=c\tau, \qquad (2)$$

falls $H(y,t)=\frac{\partial u}{\partial x}_{x=0}$ auf der YZ-Ebene gegeben ist, und aus (14) unserer Arbeit (2)

$$u(x,y,t)=-\frac{1}{\pi}\frac{\partial}{\partial x}\int_x^{ct} d\tau_1 \int_{-\sqrt{\tau_1^2-x^2}}^{\sqrt{\tau_1^2-x^2}} \frac{N(y-s,t-\tau)}{\sqrt{\tau_1^2-x^2-s^2}}\,ds;\ \tau_1=c\tau, \qquad (3)$$

falls $u(o,y,t)=N(y,t)$ ebenfalls der YZ-Ebene gegeben ist. Ferner nehmen wir als Anfangsbedingungen $G(x,y)=F(x,y)=0$ an.

Weiter setzen wir voraus, dass wir es mit elektromagnetischen Zuständen zu tun haben (alles im absoluten elektromagnetischen Mass ausgedrükt), ferner, dass die YZ-Ebene aus einem absolut reflektierenden und unendlich dünnen Schirm bestehe [siehe auch Schluss meiner Arbeit (3)].

Schon lange wusste ich, dass die tangentiale (zum Schirm, d. h. zu der YZ-Ebene) Komponente der gesamten magnetischen Kraft in der Öffnung (d. h. dort, wo der Schirm fehlt) gleich der entsprechenden Komponente der einfallenden Welle ist und zwar ganz unabhängig von der Zeit. Dieser Satz wurde im Jahre 1924 (4) auf russisch und im Jahre 1925 (5) auf deutsch publiziert.

Wir nehmen nun als die einfallende Welle eine ebene Welle senkrecht zum Schirm mit der elektrischen Kraft parallel der Z-Achse.

Ferner nehmen wir an, der Schirm habe unendlich viele und unendlich lange Öffnungen von der Breite $2b$, die zur Z-Achse parallel verlaufen, und bestehe aus Streifen von der Breite $2(a-b)$.

Demnach haben wir ein Gitter, welches unendlich lang und unendlich breit ist, mit der Gitterkonstante $2a$.

Laut dem obigen Satz wissen wir nun $\frac{\partial u}{\partial x}$ innerhalb der Öffnung, und da wir einen rein periodischen Zustand in bezug auf die Zeit vorausgesetzt haben, so erhalten wir, ebenso wie in unserer Arbeit [3], statt (1), (2), (3) die folgenden Beziehungen (wir lassen den Faktor $e^{i\omega t}$ weg):

$$\varkappa^2 u(x,y)+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \tag{4}$$

$$u(x,y)=-\frac{1}{\pi}\int_x^\infty e^{-i\varkappa\tau_1}\,d\tau_1 \int_{-\sqrt{\tau_1^2-x^2}}^{\sqrt{\tau_1^2-x^2}} \frac{H_0(y-s)\,ds}{\sqrt{\tau_1^2-x^2-s^2}} \tag{5}$$

und

$$u(x,y)=-\frac{1}{\pi}\frac{\partial}{\partial x}\int_x^\infty e^{-i\varkappa\tau_1}\,d\tau_1 \int_{-\sqrt{\tau_1^2-x^2}}^{\sqrt{\tau_1^2-x^2}} \frac{N_0(y-s)\,ds}{\sqrt{\tau_1^2-x^2-s^2}}. \tag{6}$$

Hierbei bedeutet

$$\varkappa=\frac{\omega}{c}=\frac{2\pi}{cT}=\frac{2\pi}{\lambda}\ (\lambda=\text{Wellenlänge im Vakuum}). \tag{7}$$

Siehe auch unsere Arbeit [6], in welcher u. a. die exakte Lösung für den Fall eines Drahtgitters gegeben wird.

Für die einfallende Weile setzen wir

$$u_0(x,y)=e^{-i\varkappa x}. \tag{8}$$

Wegen des obigen Satzes haben wir aus (8), innerhalb der Öffnung der Breite $2b$,

$$H_0(y)=-i\varkappa, \tag{9}$$

und auf dem Streifen ist $N_0(y)=0$.

Es wird hier alles periodisch in bezug auf die Y-Achse mit der Periode $2a$, gleich der Gitterkonstante, sein. Den Koordinatenanfang legen wir in die Mitte der Öffnung.

Wir haben hier ein gemischtes Problem vor uns, denn auf dem Streifen ist $N_0(y)=0$ und innerhalb der Öffnung ist $N_0(y)$ unbekannt. Während nun sich $H_0(y)$ innerhalb der Öffnung aus (9) bestimmt, ist es auf dem Streifen unbekannt.

Wir führen in (5) H_0 unter das Zeichen des ersten Integrals, und erhalten dann statt (5)

$$u(x,y)=\frac{1}{\pi}\int_0^\infty H_0(y-s)\,Q_0(\varkappa\sqrt{x^2+s^2})\,ds+$$

$$+\frac{1}{\pi}\int_0^\infty H_0(y+s)\,Q_0(\varkappa\sqrt{x^2+s^2})\,ds, \tag{10}$$

wobei $Q_0(x)$ die Hankelsche Funktion zweiter Gattung des Argumentes x bedeutet, multipliziert mit $\frac{i\pi}{2}$.

Infolge der erwähnten Periodizität erhalten wir aus (10):

$$u(x,y)=\frac{2}{\pi}\sum_{n=0}^\infty B_n\cos\frac{\pi n y}{a}\int_0^\infty \cos\frac{\pi n s}{a}\,Q_0(\varkappa\sqrt{x^2+s^2})\,ds. \tag{11}$$

Hieraus ergibt sich dann, wegen (29) und (30) meiner Arbeit (7):

$$\frac{2}{\pi}\int_0^\infty \cos\frac{\pi n s}{a}\, Q_0\left(x\sqrt{x^2+s^2}\right) ds =$$

$$= \left\{ \begin{array}{ll} \dfrac{ie^{-ix\sqrt{x^2-\left(\frac{\pi n}{a}\right)^2}}}{\sqrt{x^2-\left(\frac{\pi n}{a}\right)^2}} = \beta_n e^{-ix\sqrt{x^2-\left(\frac{\pi n}{a}\right)^2}} & ;\ x > \dfrac{\pi n}{a} \\ -\dfrac{e^{-x\sqrt{\left(\frac{\pi n}{a}\right)^2-x^2}}}{\sqrt{\left(\frac{\pi n}{a}\right)^2-x^2}} = \beta_n e^{-x\sqrt{\left(\frac{\pi n}{a}\right)^2-x^2}} & ;\ x < \dfrac{\pi n}{a} \end{array} \right\}. \tag{12}$$

Dies müssen wir uns nun in (11) eingesetzt denken. Weiter erhalten wir aus (12):

$$\beta_n = \frac{2}{\pi}\int_0^\infty \cos\frac{\pi n s}{a}\, Q_0(xs)\, ds, \tag{13}$$

und ferner

$$H_0(y) = \sum_{n=0}^{\infty} B_n \cos\frac{\pi n y}{a}, \tag{14}$$

$$N_0(y) = \sum_{n=0}^{\infty} B_n \beta_n \cos\frac{\pi n y}{a}. \tag{15}$$

Wegen (9) ergibt sich

$$B_0 = -\frac{ixb}{a} + C_0;\quad B_n = -\frac{2ix}{\pi n}\sin\frac{\pi n b}{a} + C_n, \tag{16}$$

wobei

$$C_0 = \frac{1}{a}\int_b^a H_0(y)\, dy;\quad C_n = \frac{2}{a}\int_b^a H_0(y)\cos\frac{\pi n y}{a}\, dy \tag{17}$$

bedeuten. Demnach haben wir laut (15):

$$N_0(y) = \left(-\frac{ix}{2} + C_0\right)\beta_0 - \frac{2ix}{\pi}\sum_{n=1}^{\infty}\frac{\beta_n}{n}\sin\frac{\pi n b}{a}\cos\frac{\pi n y}{a} +$$

$$+ \sum_{n=1}^{\infty} C_n \beta_n \cos\frac{\pi n y}{a}. \tag{18}$$

Jetzt benutzen wir den Umstand, dass auf dem Streifen $N_0(y) = 0$ ist. Wir erhalten dann aus (18):

$$N_0(y) = \left(-\frac{ix}{2} + C_0\right)\frac{b}{a}\beta_0 - \frac{2ix}{\pi^2}\sum_{n=1}^{\infty}\frac{\beta_n}{n^2}\sin^2\frac{\pi n b}{a} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{C_n\beta_n}{n}\sin\frac{\pi n b}{a} +$$

$$+ \sum_{m=1}^{\infty}\cos\frac{\pi m y}{a}\left\{\left(-\frac{ixb}{a} + C_0\right)\frac{2\beta_0}{m\pi}\sin\frac{\pi m b}{a} -\right.$$

$$\left. - \frac{2ix}{\pi a}\sum_{n=1}^{\infty}\frac{\beta_n}{n}a_{n,m}\sin\frac{\pi n b}{a} + \frac{1}{a}\sum_{n=1}^{\infty} C_n\beta_n a_{n,m}\right\}. \tag{19}$$

Hierbei bedeutet

$$a_{n,m} = \int_{-b}^{b}\cos\frac{\pi n y}{a}\cos\frac{\pi m y}{a}\, dy. \tag{20}$$

Linkerseits in (19) ist der Wert für $N_0(y)$ aus (18) zu setzen. Aus (19) können dann die C_n bestimmt werden.

Wir nehmen nun den speziellen Fall

$$b = \frac{a}{2} \tag{21}$$

an, d. h. die Streifenbreite ist gleich der Breite der Öffnung.

Wir können hier nicht die Rechnungen anführen; bemerken wir nur, dass wir zuerst

$$C_{2n+1} \cdot \beta_{2n+1} = \frac{(-1)^n}{(2n+1)^3} D + \frac{2ix(-1)^n}{\pi(2n+1)} \beta_{2n+1} \tag{22}$$

gesetzt haben, und erhielten

$$\left.\begin{aligned} B_0\beta_0 &= \frac{\pi^3}{48} D, \\ B_{2n+1} \cdot \beta_{2n+1} &= \frac{(-1)^n}{(2n+1)^3} D, \\ B_{2n+2} \cdot \beta_{2n+2} &= -\frac{(-1)^{n+1}\pi}{2(2n+2)^2} D. \end{aligned}\right\} \tag{23}$$

Da wir den Wert von $H_0(y)$ in der Öffnung, laut (9), wissen, so erhalten wir aus (14) bei $y=0$ die Beziehung

$$-ix = D\left\{\frac{\pi^3}{48\beta_0} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3\beta_{2n+1}} - \frac{\pi}{2}\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+2)^2\beta_{2n+2}}\right\}, \tag{24}$$

aus der sich dann die Konstante D bestimmt.

Für $y=a$ und $y=b$ (d. h. am Rande des Streifens) erhalten wir aus (15) die Werte Null, wie es auch sein muss. Für $y=0$ erhalten wir

$$N_0(0) = \frac{\pi^3}{16} D. \tag{25}$$

Gehen wir nun zu dem Grenzfall $\lambda \to 0$ über, wo also alle $\frac{1}{\beta_n} = -ix$ sind, so folgt aus (24) $D = \frac{16}{\pi^3}$ und $N_0(0) = 1$ bei $\lambda \to 0$.

Nehmen wir die Intensität des nullten Spektrums (d. h. des direkt durchgegangenen Lichtes) gleich eins an, so erhalten wir für die Intensitäten

$$\left.\begin{aligned} I_{2n+1} &= \left(\frac{24}{(2n+1)^3\pi^3}\right)^2 \\ I_{2n+2} &= \left(\frac{12}{(2n+2)^2\pi^2}\right)^2 \end{aligned}\right\}, \tag{26}$$

berechnet nach dem Quadrat der elektrischen Kraft.

So haben wir, z. B., $I_0 = 1$; $I_1 = 0.5910$; $I_2 = 0.0924$; $I_3 = 0.0008$; $I_4 = 0.0058$.

Zum Schluss sei noch auf die Arbeiten von G. Jaffé ([8]), ([9]) und E. Fischer ([10]) hingewiesen.

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PHYSIQUE MATHÉMATIQUE

SUR LA PROPAGATION DE LA CHALEUR DANS UN MILIEU À PLUSIEURS COUCHES

Par N. MALKIN

(Présenté par A. N. Kryloff, de l'Académie, le 25. IV. 1938)

Dans le cas d'un milieu plan et parallèle ce problème a été résolu d'une manière assez compliquée [par la méthode «of wave-trains» et par les intégrales de contour, Robertson (1) et S. S. Kovner (2)]. A. N. Krylov (3) dans le cas d'un cylindre à trois couches se sert de la méthode ordinaire de Fourier, mais pour éviter la détermination des poids des fonctions fondamentales, il applique dans la suite la méthode des plus petits carrés. A. N. Tikhonoff (4) se borne dans le cas d'un milieu plan ou sphérique à l'examen de l'approximation de la solution trouvée relativement à la solution stationnaire *. Cependant à l'aide d'une substitution régulatrice (5) les singularités sur les surfaces de séparation entre les couches peuvent être anéanties et, par conséquent, dans tous les cas indiqués les poids peuvent être trouvés sans difficultés; d'après quoi le problème se résoud très simplement par la méthode de Fourieur.

En prenant l'équation de la conduction thermique

$$K\delta \frac{\partial v}{\partial t} = \operatorname{div}(\eta \operatorname{grad} v), \tag{1}$$

où v, K, δ et η désignent respectivement la température, la chaleur spécifique, la densité et le coefficient de la conductibilité du milieu, nous supposerons que le milieu en question est divisé en plusieurs couches homogènes par les surfaces ayant toutes une des coordonnées curvilignes q_1 constante.

Supposons, pour simplifier, que la température v ne dépend que d'une seule coordonnée q_1 (notons que le problème se résoud aussi bien dans le cas général) et posons $v = T(t)\,Q(q_1)$, puis $\frac{T_1}{T} = -\lambda^2$.

Mettons (1) sous la forme

$$\frac{d}{dq_1}\eta \frac{A_2 A_3}{A_1}\frac{dQ}{dq_1} = -K\delta A_1 A_2 A_3 \lambda^2 Q, \tag{2}$$

où A_1, A_2 et A_3 sont les coefficients de l'expression du carré de la différentielle de l'arc $ds^2 = A_1^2 dq_1^2 + A_2^2 dq_2^2 + A_3^2 dq_3^2$.

* La solution exacte n'est donnée que pour le milieu homogène (K=const, δ=const, η=const).

Après avoir résolu ces équations pour chaque couche séparément, on réunit les solutions obtenues * en une seule à l'aide des conditions frontières homogènes ou bien du type ordinaire, à savoir:

$$Q \text{ et } \eta \frac{dQ}{A_1 dq_1} \text{ sont continus à la frontière de deux couches,} \quad (3)$$

ou bien avec la perte de chaleur, comme on trouve chez A. N. Kryloff, dans ce cas $\eta \frac{dQ}{A_1 dq_1} = bQ$, sur les frontières extérieures (car il est toujours possible de ramener le problème à la condition que la température du milieu extérieur soit égale à zéro). Alors il est facile à vérifier que les fonctions fondamentales trouvées sont mutuellement orthogonales avec le poids discontinu $K\delta A_1 A_2 A_3$ et l'on aura:

$$\int K\delta A_1 A_2 A_3 Q_m Q_n dq_1 = 0, \quad (4)$$

où l'intégrale est étendue à tout l'intervalle de valeurs que peut prendre q_1; Q_m et Q_n sont les solutions correspondantes à λ_m^2 et λ_n^2.

Pour cela il faudra former une intégrale d'une certaine combinaison de deux expressions de la forme (2) et intégrer dans tout l'intervalle dans les conditions (3), ou bien diviser l'intégrale en parties dans le cas de perte de chaleur vers l'extérieur.

Pour les systèmes de coordonnées plan et parallèle, cylindrique ou sphérique, les produits $A_1 A_2 A_3$ seront respectivement égaux à 1, r, $r^2 \sin\varphi$, c'est-à-dire, les poids seront $K\delta$, $K\delta r$ et $K\delta r^2$. Par conséquent, les fonctions $p(r)$, $q(r)$, $s(r)$ des formules (31) de A. N. Kryloff sont égales à une même grandeur discontinue $K\delta r$.

La partie de recherche la plus difficile, à savoir le calcul des racines du déterminant caractéristique $\Delta(\lambda) = 0$, peut être executée dans le cas d'un câble cylindrique, si au lieu du déterminant on examine les équations qui y aboutissent.

En prenant les solutions particulières [cf. (20) A. N. Kryloff] **:

$$\left.\begin{aligned} v_1 &= AI_0\left(\frac{\lambda}{\sqrt{k_1}} r\right) \qquad\qquad = AI_1, \\ v_2 &= CI_0\left(\frac{\lambda}{\sqrt{k_2}} r\right) + DN_0\left(\frac{\lambda}{\sqrt{k_2}} r\right) = CI_2 + DN_2, \\ v_3 &= EI_0\left(\frac{\lambda}{\sqrt{k_3}} r\right) + FN_0\left(\frac{\lambda}{\sqrt{k_3}} r\right) = EI_3 + FN_3, \end{aligned}\right\} \quad (5)$$

et en notant que d'après Hütte pour le cuivre $p_2 = \sqrt{K_2 \delta_2 \eta_2}$ est approximativement égale à 50 et p_1 et p_3 à 0.2, on peut, suivant les indications de A. N. Kryloff, négliger les rapports $\frac{p_1}{p_2}$ et $\frac{p_3}{p_2}$, alors les conditions différentielles de la forme (3) (qui sont justes aussi bien dans le cas de perte de chaleur à l'extérieur):

$$\eta_2 \frac{dQ_2}{dr} = \eta_1 \frac{dQ_1}{dr} \text{ pour } r = a \quad \text{et} \quad \eta_2 \frac{dQ_2}{dr} = \eta_3 \frac{dQ_3}{dr} \text{ pour } r = b,$$

---

* Ces solutions seront exprimées par des fonctions trigonométriques pour les couches planes, par des fonctions de Bessel pour les couches cylindriques et sphériques, par les fonctions plus compliquées pour les couches en forme d'un ellipsoide.

** Ici ne figurent que les fonctions de Bessel et de Neuman du rang nul. Pour cela afin de simplifier l'écriture, les indices du rang nul ne sont pas écrits dans les derniers membres de droite.

seront de la forme

$$\begin{aligned} CI'_{2,a} + DN'_{2,a} &= 0, \\ CI'_{2,b} + DN'_{2,b} &= 0. \end{aligned} \tag{6}$$

La condition de leur compatibilité sera

$$I'_{2,a} N'_{2,b} - I'_{2,b} N'_{2,a} =$$
$$= I_1\left(\frac{\lambda}{\sqrt{k_2}} a\right) N_1\left(\frac{\lambda}{\sqrt{k_2}} b\right) - I_1\left(\frac{\lambda}{\sqrt{k_2}} b\right) N_1\left(\frac{\lambda}{\sqrt{k_2}} a\right) = 0.$$

En se servant des expressions asymptotiques des fonctions de Bessel

$$I_0(x) \sim -N_1(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4}\right),$$
$$I_0(x) \sim N_0(x) \sim \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4}\right),$$

il est facile à trouver les racines de cette équation, qui sont

$$\lambda_n \sim \frac{\pi \sqrt{k_2}}{b - a} n \qquad (n = 1, 2, 3, \ldots). \tag{7}$$

Notons que les premières racines de cette équation peuvent être trouvées dans les tables(6).

D'après cela il est facile à exprimer tous les coefficients D, A, E et F au moyen de C, ce qui sera précisément la solution du problème dans la première approximation.

La seconde approximation, si l'on en aura besoin, se réduira à la résolution d'un système de cinq équations linéaires aux cinq inconnues.

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1938. Volume XX, № 2—3

PHYSICS

POSITRON SPECTRUM OF AN ACTIVE THORIUM DEPOSIT

By A. I. ALIKHANOV and V. P. DŽELEPOV

(Communicated by A. F. Joffe, Member of the Academy, 7.V.1938)

The positron spectra emitted by thin walled radon ampules and by active thorium deposits have already been studied by Alikhanov, Alikhanian and Kozodaev (1).

These authors had at their disposal only a very weak source of active thorium deposit (1 mg-equivalent) so that the accuracy of the individual measurement could not be great. The positron distribution curve given in their paper was obtained by combining and averaging a large number of separate curves.

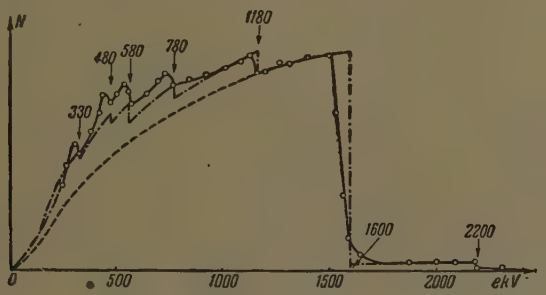

Fig. 1.

Since a stronger positron source (10 mg-equivalents) as well as a considerably improved apparatus are now available, we repeated the investigation of this spectrum. The results of these measurements are shown in Fig. 1.

The experimental curve in Fig. 1 is again the sum of several separate curves or parts of curves which were however obtained with considerably greater statistical accuracy. The statistical accuracy of the curve given is about 2—3%. The newly obtained curve leads us to some highly important conclusions. Besides the sharp break in the curve at an energy value of 1600 ekV, this curve shows six new, considerably smaller but quite well marked discontinuities. By using the equation

$$h\nu = 2\,mc^2 E_l$$

(where E_l is the energy corresponding to the position of the discontinuity), we find that besides the γ-line at 2600 ekV, the active thorium deposit emits γ-rays with energies 1350 ± 20 ekV, 1500 ± 20 ekV, 1600 ± 20 ekV, 1800 ± 20 ekV, 2200 ± 20 ekV, and 3200 ± 40 ekV.

The table shows the intensities of these energies relative to the intensity of the γ-line 2620 ekV on the assumption that all these lines are of quadrupolar origin.

Number of the γ-line	Energy of the γ-line	Fraction of the positrons on the given line (%)	Intensity of the line (%)
1	1350	0.6	10.0
2	1500	0.9	6.5
3	1600	2.2	11.0
4	1800	2.1	6.5
5	2200	6.6	10.0
6	2620	100.0	100.0
7	3200	3.3	2.5

The table shows: 1) that only 84% of all the positrons from the active thorium deposit are due to the γ-line 2620 ekV; 2) that only 2/3 of all the hard (> 1000 ekV) quanta emitted by the active thorium deposit have an energy of 2620 ekV.

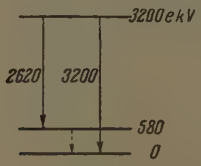

Fig. 2a.

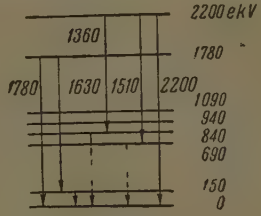

Fig. 2b.

From the theoretical positron distribution curves for the separate γ-lines, as drawn on the basis of the magnitudes of the discontinuities, we were able to construct the total theoretical positron spectrum of the active thorium deposit. By comparing this (Fig. 1) with the experimental spectrum, we find that the two practically coincide with one another. This means that the whole positron spectrum of the active thorium deposit may be completely explained by inner conversion of γ-rays on the negative energy levels.

By using the active thorium deposit γ-ray spectrum so obtained together with the results of the analysis of the long range groups of α-particles, we can construct a scheme of the energy levels of the ThD and ThC' nuclei. The ThD energy level scheme proposed by Ellis and Mott[2] is confirmed by the presence of the line 3200 ekV and is shown in Fig. 2a.

This proposed energy level scheme of the ThC' nucleus (Fig. 2b) is confirmed from yet another point of view; for the low energy γ-lines (150, 690 and 840 ekV) which follow from this scheme actually appear in the natural β-spectrum of Th (B+C+C'')[3].

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PHYSICS

POSITRON SPECTRUM EMITTED BY LEAD ON IRRADIATION WITH ThC'' γ-RAYS

By A. I. ALIKHANOV and V. P. DŽELEPOV

(Communicated by A. F. Jôffe, Member of the Academy, 7.V.1938)

The first attempts to study the true positron spectrum produced in a heavy element irradiated by monochromatic γ-rays were made by Alikhanov, Alikhanian and Kozodaev([1]). Their results showed that the spectrum of such positrons is asymmetric and that the majority of the positrons have an energy greater than

$$\frac{h\nu - 2mc^2}{2}.$$

Due to the small intensity of the γ-ray source (an active thorium deposit), they could obtain no great accuracy in their experiments. These experiments were repeated under improved experimental conditions and with a more powerful thorium deposit.

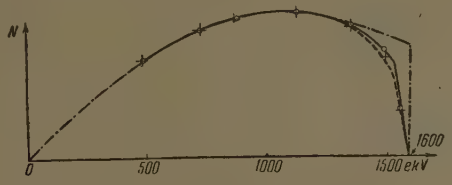

Fig. 1.

The source of the γ-rays was placed in a graphite thimble with 2—2.5 mm walls covered with thin lead foil.

The positron spectrum formed in the foil was found by subtracting the positron spectrum of the graphite from the positron spectrum graphite + lead. From the distribution curve thus obtained was subtracted the part of the positrons produced by γ-rays with an energy of 2200 kV, the most powerful of the γ-rays we found from an active thorium deposit. Experiments were made using both 30 and 17 mg/cm² thicknesses of lead foil.

The positron distribution curves obtained with these two foils are shown in Fig. 1.

The difference between them is very small, which indicates that for these thicknesses the true positron spectrum is but very little distorted by the finite thickness of the absorbent. The same figure shows the curve cal-

culated by Egert and Holm (dotted curves). This is in excellent agreement with the experimental curve except in those regions of the curve which are in the immediate nieghbourhood of the spectral limit. It should be noted, however, that it is just in this region of the spectrum that distortions due to the finite thickness of the absorbent must be particularly marked.

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PHYSICS

IRREVERSIBLE AND REVERSIBLE CHANGES OF MAGNETIZATION IN FERROMAGNETICS UNDER TENSION AND MODE OF INCREASING MAGNETIC FIELD

By E. KONDORSKY

(Communicated by S. I. Wawilow, Member of the Academy, 17.V.1938)

In order to obtain more detailed information about the magnetic properties of Fe—Ni alloys under tension(1) we carried out, by means of an astatic needle magnetometer*, the measurements of 1) the magnetization curves $I(H)$; 2) curves of the residual magnetism as a function of the field, $I_r(H)$; 3) curves of the reversible susceptibility [according to R. Gans(3)] as a function of the field $\chi_r(H)$. These measurements were carried out for a Fe—Ni alloy with 15% Ni both without stress and with a tensile stress $F = 74.4$ kg/mm^2. In carrying out these measurements it was found that the difference usually observed between the magnetization curves with and without switching over the current became many times greater when the stress was applied to the specimen. This effect was the subject of a special study.

To elucidate this point the following two methods were used:

a) As the field increased, the magnetizing current was continuously switched over, i. e. the field continuously changed its sign while increasing.

b) The current was not switched over at all, i. e. the field increased steadily without changing its sign. Curves $\chi(H)$ and $\chi_r(H)$ represented in Fig. 1 and curves $I_r(H)$ represented in Fig. 2 are obtained for specimens in the stressed and unstressed states by the two methods mentioned above. The curves for specimens in the unstressed state, obtained by methods a) and b), are in good agreement. In the case of stressed specimens this agreement takes place only if the demagnetization is carried out under tension (by means of switching over of the magnetizing current and gradual decrease of its value down to zero). It will be seen from Fig. 1 that when the sign of the field is continuously changing as the field is increased, the residual magnetism I_r remains close to zero until a certain value of the field H_0 is reached.

* The magnetometer used for measurements was described by Akulov and Bytchkov (3) who studied by means of this apparatus the effect of elastic tensions upon the initial susceptibility of Fe, Ni and Co.

In this case we see (Fig. 1) that in a field of lower intensity than the value of H_0 the susceptibility remains, within the limits of experimental error, equal to the reversible susceptibility.

When the field was increasing without changing its sign (method b), after demagnetization in the unstressed state, the residual magnetism in the case of stressed specimens appeared already in the weakest fields and attained very large values in fields less than H_0. This shows that the irreversible processes involved in magnetization may take place in fields of much lower intensity than H_0 or Sixtus and Tonks'(1) critical field. The initial susceptibility of the specimens which were demagnetized without

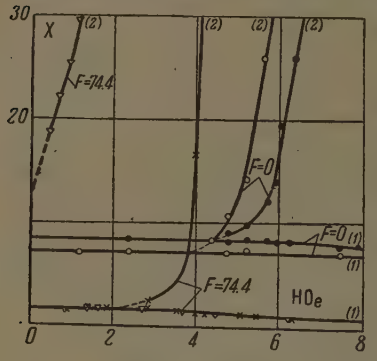

Fig. 1. Susceptibilities of Fe—Ni alloy with 15% Ni: *1*—reversible susceptibility, *2*—susceptibilities.

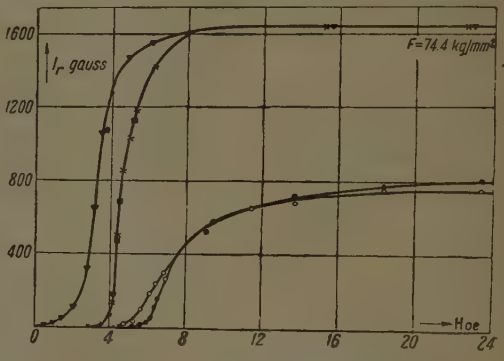

Fig. 2. Residual magnetism of Fe—Ni alloy with 15% Ni.

—○—○—○— $F = 0$ before loading —●—●—●— $F = 0$ after loading	The specimen was demagnetized without load (in unstressed state).
—▽—▽—▽— $F = 74.4$ kg/mm²	The specimen was demagnetized without load. Switching over was not carried out. The magnetic field does not change its sign as it is increased.
—×—×—×— $F = 74.4$ kg/mm²	The specimen was demagnetized without load; continuous switching over was carried out as the field was increased.
—■—■—■— $F = 74.4$ kg/mm²	The specimen was demagnetized under tension.

external stress and then stressed and measured by method b) is to a considerable extent irreversible. The irreversible initial susceptibility increases as the tension is increased whilst the reversible one diminishes. When the tension F is 74.4 kg/mm², the initial reversible susceptibility is about one fifth of the corresponding value in the unstressed state. As will be seen from Fig. 1, the value of the initial reversible susceptibility does not depend on the manner of increasing the field. If a stressed specimen is magnetized and thereupon demagnetized under stress* (after the method described above), the curves obtained using method a) and method b) do agree both with each other and with the curve previously obtained by method a) (after demagnetization without external stress). To obtain a curve with an increased initial suscep-

* The effect of demagnetization under stress upon the initial susceptibility of Fe-crystals was simultaneously studied by M. V. Degtiar. The results of his investigation will appear before long.

tibility it is necessary to take off the stress from the specimen, demagnetize it and after having stressed it anew, to increase the field without changing the sign of the magnetizing current (according to method b).

One of the possible explanations of the dependence of Ir and χ upon the manner of increasing the field consists in the following: on demagnetization without external stress domains are formed, the spins of which have various directions depending on the directions of the internal stresses. After a sufficiently strong tension has been applied, the line of extension becomes the axis of easy magnetization, and the spins of the domains become parallel to this axis (and parallel or antiparallel to each other). Using the condition of minimum magnetic and elastic energies we may obtain, from Bloch's(4)* expression for the surface energies of domains, the value H_c of the magnetic field in the volume surrounding the boundary of domains with opposite spins, viz. the value of the field at which this boundary is set in motion and one of the domains absorbs the other. Calculation(6) leads to the following value of this field:

$$H_c = \frac{\lambda' \delta}{4 I_s \cos \varphi} \frac{\partial \sigma_s}{\partial n}, \qquad (1)$$

where $\delta = \sqrt{\frac{J}{K + \lambda' \sigma_s}}$ (according to Bloch's theory δ is the width of the boundary between the domains), J is the exchange integral, K is the constant of magnetic anisotropy, a is the lattice parameter, σ_s is the stress in a direction parallel to that of the spins of the domains, ∂n is the displacement of the boundaries in the direction of the normal to the surface, λ' is a quantity of the same order as magnetostriction on saturation and φ is the angle between the spins of the engulfed domain and the direction of the magnetic field. It follows from (1) that changes in the direction of the spins involve a variation in the value of H_c. It is not only the increase of $\cos \varphi$ or decrease of δ which account for the variation of H_c. The latter is also due to the change of $\frac{\partial \sigma_s}{\partial n}$, because the value of σ_s varies with the direction of spins. Here, owing to the «equalizing» effect of the external stress, $\frac{\partial \sigma_s}{\partial n}$ diminishes (or even become equal to zero) in some parts of the specimen, and many of the boundaries between the domains prove to be in a state of unstable equilibrium. The irreversible change of magnetization should therefore begin at weaker fields in this case, than it does when demagnetization of the specimens is carried out under tension and domains are formed which are stable under given conditions.

If the current is continuously switched over as the field is increased (method a), the process of absorption of unstable domains takes place in both directions, and equal quantities of stable domains with opposite spins are formed. Therefore no residual magnetism appears in this case until a definite value of the field H_0 is attained.

It follows from the above that demagnetization in the unloaded state and subsequent application of a load may be used as a method of obtaining such initial states in hard materials with positive magnetostriction as are only possible after annealing when ordinary methods of demagnetization are used. The high values of magnetic susceptibility obtained by this method in some hard materials are an evidence of the magnetization in the latter

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* Bloch's expression for the surface energy of domains found experimental support in the work of Dorring(5).

being possible in weak fields. Thus, the difference between hard and soft materials is that in the former the conditions are usually not available for magnetization to begin in weak fields.

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PHYSIQUE

LA MÉTHODE D'AUGMENTATION DE TENSION DU COURANT CONTINU À L'AIDE DES CONDENSATEURS

Par V. I. LIKHOFF et V. I. PAVLOFF

(Présenté par S. I. Wawilow, de l'Academie, le 21.V.1938)

Le problème de la production de hautes tensions pour les besoins aussi bien techniques que de la physique de nos jours est très actuel. Peu de méthodes existantes sont en mesure de produire des tensions supérieures à un million de volts. Mais tandis que les unes ne donnent que des impulsions à courte durée, les autres produisent des courants à l'intensité minime.

Dans cet article on trouvera la description de l'installation aux condensateurs, permettant de produire le courant continu de haute tension et d'une puissance suffisante.

L'installation décrite est basée sur la transformation de la tension moyennant la charge des condensateurs réunis en série. Mais cette méthode se distingue de toutes les autres du même genre[1]) par ce fait que la charge des condensateurs réunis en série s'effectue par l'intermédiaire d'un système de condensateurs auxiliaires. Les condensateurs auxiliaires servent, par conséquent, de lien entre le redresseur chargeant et la colonne des condensateurs réunis en série.

Chaque condensateur auxiliaire chargé représente un réservoir de l'énergie électrique n'étant plus réuni à l'installation qui produit la charge. Le condensateur auxiliaire n'a pas besoin pendant toute la période du cycle, c'est-à-dire pendant le déplacement de la borne de la source chargeante sur tous les contacts de la colonne et inversement, de l'arrivée de l'énergie de l'extérieur. Étant bien isolé, il peut prendre les potentiels de la colonne à laquelle il se réunit.

Pour réaliser ce système de commutation la colonne des condensateurs réunis en série est placée à l'intérieur d'un gros disque tournant; les conducteurs dérivant des parties distinctes du condensateur aboutissent aux contacts situés sur la circonférence (fig. 1).

Sur la coupe verticale donnée nous avons un disque a et la colonne b de condensateurs réunis en série, placée à l'intérieur. Les conducteurs dérivant des sections de la colonne sont réunis avec les contacts c situés sur la circonférence du disque. Le disque avec le condensateur sectionné placé à son intérieur est mis en mouvement, et dans sa rotation il peut toucher par ses contacts les balais des condensateurs auxiliaires d, f. La tension du redresseur chargeant est transmise, par l'intermédiaire de l'axe et des contacts qui se frottent, aux bandes e situées de même sur la circonférence. Ces bandes

sont placées sur la même hauteur que les contacts de la section intérieure, et en faisant tourner le disque on a le moyen de charger tous les condensateurs auxiliaires chacun à son tour.

La fig. 2 montre l'installation vue du haut, la légende étant la même que pour la fig. 1. Les indices des contacts correspondent au rang des condensateurs dans la colonne, à savoir: le numéro *1* désigne le condensateur situé tout en bas, l'indice *n*—le condensateur situé tout en haut. On voit sur la fig. 2 le nombre double des contacts pour chaque section; leur destination sera expliquée dans la suite.

Dans l'installation décrite, la colonne des condensateurs est composée des condensateurs électrolytiques réunis en série. Les condensateurs électrolytiques ont été employés vu leur petite dimension et leur grande capacité.

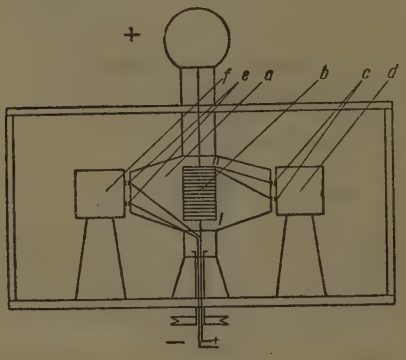

Fig. 1.

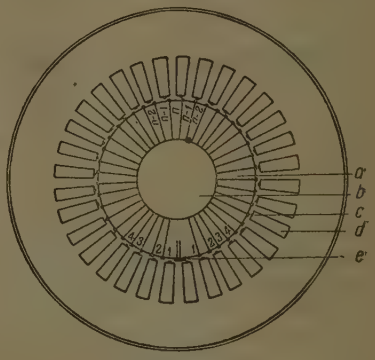

Fig. 2.

Le condensateur électrolytique est composé d'une plaque couverte d'une pellicule oxydée, servant d'anode, et d'un cathode. L'espace qui les sépare est rempli d'électrolyte. Pour cathode on prend généralement une plaque d'aluminium qui est destinée à faire venir le courant à l'électrolyte, car le rôle de condensateur joue, proprement dit, la plaque d'anode et la couche de l'électrolyte voisine à la plaque. La pellicule oxydée représente un diélectrique. Un condensateur électrolytique peut fonctionner grâce aux propriétés de la pellicule oxydée pour les tensions non supérieures à 500—700 V.

Ainsi une colonne de condensateurs réunis en série pour un million de volts doit comprendre au moins 2000 condensateurs distincts. Un tel nombre de condensateurs distincts causerait de grandes difficultés quant à leur disposition, connexion et isolation l'un de l'autre. Pour cette raison on a été conduit à créer un nouveau type du condensateur électrolytique à plaques, destiné à hautes tensions.

Le condensateur à plaques est composé de plaques d'aluminium oxydées se succédant avec des couches intermédiaires de papier trempé d'électrolyte. Les plaques ne sont oxydées que d'un seul côté, car ce n'est que ce côté qui joue le rôle d'anode, l'autre servant de cathode du condensateur voisin. Par conséquent, au lieu de condensateurs distincts réunis en série, nous avons une colonne de condensateurs au nombre de plaques deux fois plus petit.

On couvre la batterie de tous les côtés d'une masse isolante. La hauteur d'une telle batterie pour une installation d'un million de volts ne sera que de 30 à 70 cm. Comme un condensateur électrolytique ne peut supporter que la tension d'ordre de 500 V, il vaut mieux de faire des dérivations non pour chaque condensateur séparément, mais pour un groupe de condensateurs suivant la tension que nous nous proposons de transformer. Par exemple, pour

élever la tension de 50 kV à un million de volts, nous aurons besoin d'une batterie à 2000 plaques avec les fils dérivant de chaque centième plaque.

Les condensateurs auxiliaires seront de même électrolytiques et réunis en série. Lorsque le disque tourne, à un moment donné les balais d'un certain condensateur touchent les bandes chargeantes. Puis la même opération a lieu pour le condensateur suivant, etc. Après les bandes, ce sont les contacts de la section inférieure de la colonne qui viendront toucher les balais du condensateur chargé à l'instant. Le condensateur auxiliaire lui cédera une partie de sa charge. Enfin viendront les contacts de la section supérieure de la colonne (fig. 2, désigné par n).

Pendant le temps du déplacement des contacts de la colonne sur les contacts des condensateurs auxiliaires, ces derniers vont prendre successivement les potentiels correspondant aux contacts de la colonne. Comme la borne inférieure de la colonne est réunie à la terre, le condensateur aura d'abord le potentiel relatif à la terre égal à zéro, puis va prendre une suite de valeurs jusqu'à la valeur maximum donnée par nos condensateurs réunis en série.

Mais le condensateur auxiliaire, tout en étant bien isolé des autres parties du système, possède tout de même une certaine capacité C' par rapport au système.

Pour les tensions d'ordre d'un million de volts quelques dizaines de centimètres pourront jouer un rôle essentiel. Si immédiatement apres le condensateur du rang n nous avions réuni notre condensateur auxiliaire aux bandes chargeantes, nous aurions diminué la charge de la colonne principale de la charge $C'U$. Le nombre de telles charges est égal au nombre de condensateurs auxiliaires, c'est-à-dire à quelques dizaines.

La somme de leurs capacités peut atteindre une valeur très appréciable. Les dérivations secondaires dont il a déjà été question servent précisément pour enlever et utiliser ce genre d'énergie.

Si pendant le déplacement des balais des condensateurs auxiliaires depuis le premier jusqu'à n-ème contact nous chargeons la collonne par le condensateur auxiliaire, ce dernier lui même fait élever son potentiel par rapport à la terre. La seconde partie du cycle de n à 1 a pour but la décharge de cette capacité «extérieure» du condensateur auxiliaire. La décharge C' se fera au moyen des contacts successifs avec les dérivations des sections intérieures.

De plus, il est désirable que les deux balais du condensateur auxiliaire ne touchent pas les contacts des sections intérieures de la colonne pendant la seconde partie du cycle. Pendant le déplacement du contact n au contact 1 la capacité «extérieure» C' du condensateur auxiliaire va perdre sa charge, en rendant l'énergie à la colonne des condensateurs.

Passons maintenant à la question de la puissance et de l'intensité du courant que pourrait donner une installation pareille.

La puissance et l'intensité peuvent être approximativement calculées à partir des données suivantes: la capacité des sections de la colonne, la tension pour laquelle est faite la colonne, le nombre de tours du disque et le nombre des condensateurs auxiliaires.

De plus, il faudra convenir sur la façon dont on va décharger nos condensateurs réunis en série. Il est évident qu'il n'est pas avantageux de les décharger à partir du zéro, car ce n'est pas des impulsions, mais de hautes tensions à grandeur, autant que possible constante, que nous voulons obtenir.

Par conséquent, nous devrons décharger notre colonne en partie relativement petite de sa charge primitive.

Supposons que nous allions décharger $^1/_{10}$ ou bien $^1/_5$ de la charge primitive. Dans ces cas les intensités que donnera l'installation seront de même différentes. Évidemment le courant sera d'autant plus grand que la partie

dont nous allons décharger le condensateur sera plus grande. Désignons la grandeur, caractérisant le degré de décharge, par K.

L'intensité, pour les valeurs de K suffisamment petites, s'exprimera de la manière suivante:

$$I = CU\,np\,K,$$

où I désigne l'intensité du courant en ampères; C—la capacité de la colonne des condensateurs réunis en série en farads; n—le nombre de tours du disque en seconde; p—le nombre des condensateurs auxiliaires; K—le coefficient de décharge.

Posons $U = 10^6$V.

La capacité totale C pour 2000 plaques et pour la capacité de chacune d'elles égale à 5 mF sera égale à $2.5 \cdot 10^{-9}$ F. On peut prendre n égal à 5 tours par seconde; p, le nombre des condensateurs auxiliaires, sera 50; $K = \frac{1}{10}$.

On aura alors pour intensité

$$I = CU\,np\,K \approx 60 \cdot 10^{-3}\,\text{A}$$

et pour puissance

$$P = 60\,\text{kW}.$$

Pour la vérification du principe exposé, ou plus exactement de quelques éléments de construction, a été construit un petit modèle pour la tension de 80—100 kV. La tension primaire était de 7 kV, par conséquent le coefficient de transformation était voisin de 12, ce qui a été pratiquement réalisé.

La hauteur de la colonne composée de 220 condensateurs réunis en série était égale à 8 cm. La colonne était divisée en sections, chaque section comprenant 16 condensateurs. Pour juger des dimensions de l'installation, indiquons que le rotor était placé sur le disque d'un moteur synchrone d'un phonographe faisant 78 tours par minute.

Après tout ce qui a été exposé, on peut considérer que ce principe permettra de construire les installations pour des millions de volts, en obtenant des intensités suffisamment grandes.

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PHYSICS

ON THE NON-ELASTIC DISPERSION OF PHOTONS ABOUT NUCLEI WITH PAIR-PRODUCTION

By M. MARKOV

(Communicated by S. I. Wawilow, Member of the Academy, 26.V.1938)

As is well known, the method of disturbance gives in problems of this kind an expansion into a power-series with respect to the fine-structure constant.

The factor $\left(\frac{e^2}{\hbar c}\right)^n$, necessarily appearing in the expressions for the cross-section in the case of processes of higher order between electrons and light-quanta, considerably diminishes (since n increases) the numerical value of the cross-section.

B. Kockel has, however, pointed out(1) that in the case of great energies in the expressions for the cross-section factors may appear which compensate the effect of the factor $\left(\frac{e^2}{\hbar c}\right)^n$.

From this point of view it is of interest to analyze (for great $h\nu$) the non-elastic dispersion of a photon about a nucleus accompanied by pair-production. Following Kockel, it could be expected that for great energies of the photon this effect will turn out to be as probable as the effect of the simple pair-production under the influence of the photon. This is, however, not the case.

In fact, Kockel obtains for the process of pair-production under the influence of two light-quanta and a simultaneous emission of a third quantum (process of the third order) the following expression for the cross-section:

$$\Phi^{III} = 24\left(\frac{e^2}{mc^2}\right)^2 \frac{e^2}{\hbar c} \frac{1}{\left(\frac{h\nu}{mc^2}\right)^2} \lg^3 \frac{h\nu}{mc^2} \qquad (h\nu \gg mc^2). \tag{1}$$

This expression is obtained in a system of co-ordinates, in which the impulses of the photons are equal and opposite. With the same approximation and in the same system of co-ordinates the corresponding expression for the formation of a pair under the influence of two light-quanta [process of the second order—Breite-Willer's expression(2)] has the form:

$$\Phi^{II} = 2\pi\left(\frac{e^2}{mc^2}\right)^2 \frac{1}{\left(\frac{h\nu}{mc^2}\right)^2} \lg \frac{h\nu}{mc^2} \qquad (h\nu \gg mc^2). \tag{2}$$

The comparison of the expressions (1) and (2) shows that for great energies ($h\nu \gg mc^2$) expression (1) is comparable with or may even be greater than expression (2):

$$\frac{\Phi^{III}}{\Phi^{II}} = \frac{12}{\pi} \frac{e^2}{\hbar c} \lg^2 \frac{h\nu}{mc^2}. \tag{3}$$

We proceed now to estimate the effect of the non-elastic dispersion of photons about a nucleus by the method of Williams.

Let the energy of the falling photon be $h\nu' = \xi mc^2$. Let us consider the system of co-ordinates in which the nucleus moves towards the photon with the velocity determined by

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \frac{1}{2}\xi. \tag{4}$$

In this system (the S-system) the field of the nucleus may be replaced by an equivalent field of the photons ([3]):

$$N(\nu)\, d\nu = \frac{2}{\pi} \alpha z^2 \lg \xi \frac{mc^2}{h\nu} \frac{d\nu}{\nu}, \tag{5}$$

where α is the fine-structure constant, z—the charge of the nucleus and $N(\nu)\, d\nu$—the number of photons in the interval ν, $\nu + d\nu$.

In the same system of co-ordinates the energy of the falling photon is

$$h\nu = mc^2. \tag{6}$$

The non-elastic dispersion of the photon about a nucleus accompanied by pair-production may be considered as a result of Kockel's process proceeding under the influence of a photon falling upon the nucleus and photons of the equivalent field (5). But it is known that the pair-production under these circumstances proceeding according to the Breite-Willer's formula (2) causes the production by the photon of a pair in the field of the nucleus. Using (1), (2) and (5), we can calculate both these effects.

First of all let us transform the expressions (1) and (2) into the system S. The expression (1) may be written in the form:

$$\Phi^{III} = 3 \left(\frac{e^2}{mc^2}\right)^2 \frac{e^2}{\hbar c} \frac{1}{\left(\frac{h\nu}{mc^2}\right)^2} \lg^3 \left(\frac{h\nu}{mc^2}\right)^2. \tag{7}$$

It is easy to see that the product of the frequencies ν_1 and ν_2 of two photons, having oppositely directed impulses, is invariant with respect to the transformation of Lorenz:

$$\nu_1 \nu_2 = \nu_1' \nu_2' = \nu^2. \tag{8}$$

Using (8) we may rewrite (7) in any system of co-ordinates, and introducing new denotations $\left(\frac{h\nu}{mc^2} \to \nu\right)$ we obtain:

$$\Phi^{III} = 3 \left(\frac{e^2}{mc^2}\right)^2 \frac{e^2}{\hbar c} \cdot \frac{1}{\nu_1' \nu_2'} \lg^3 \nu_1' \nu_2'. \tag{9}$$

Changing to system S and putting $\nu_1 = 1$ (i. e. $h\nu = mc^2$), we get

$$\Phi^{III} = 3 \left(\frac{e^2}{mc^2}\right)^2 \frac{e^2}{\hbar c} \frac{1}{\nu} \lg^3 \nu. \tag{10}$$

The expression (2) is transformed similarly:

$$\Phi^{II} = \pi \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{\nu} \lg \nu. \tag{11}$$

Using (2), we get the cross-section for the pair-production (in the field of the nucleus) by the photon in the form:

$$\sigma^{II} = \int_1^{\eta} \Phi^{II} N(\nu)\, d\nu = 4\alpha z^2 \left(\frac{e^2}{mc^2}\right)^2 \lg \xi. \qquad (12)$$

This result differs from the result of Williams by the factor 9/7. This is due to the fact that unlike Williams we used the asymptotical ($\nu \gg 1$) expression (2) of Breite-Willer's formula. As we see, no great error arises, if we do so.

For Φ^{III} we have only an asymptotical expression. Using this expression (3), we obtain the cross-section for the non-elastic dispersion of a photon about a nucleus accompanied by pair-production in the form:

$$\sigma^{III} = \int_1^{\eta} \Phi^{III} N(\nu)\, d\nu = \frac{36}{\pi} \alpha^2 z^2 \left(\frac{e^2}{mc^2}\right)^2 \lg \xi + \frac{1}{\eta} f(\lg \eta), \qquad (13)$$

where $f(\lg \eta)$ is a polynomial of the third degree in $\lg \eta$. We see that for great energies ($\eta \gg 1$) the expression (13) becomes

$$\sigma^{III} = \frac{36}{\pi} \alpha^2 z^2 \left(\frac{e^2}{mc^2}\right)^2 \lg \xi,$$

and the ratio $\sigma^{III} : \sigma^{II}$ is here, unlike Kockel's case, independent of the energy:

$$\frac{\sigma^{III}}{\sigma^{II}} \approx \alpha.$$

It could seem that in our case we are likely to get a result similar to that of Kockel [3], because the calculations of our effect are based upon the effect Φ^{III} and Φ^{II}, which are just those compared by Kockel. But in the effect under consideration the processes Φ^{III} and Φ^{II} play only an insignificant rôle when $\eta \gg 1$ [the integrals (12) and (13) rapidly converging], whereas Kockel's remark becomes essential only for large η.

A result similar to ours is to be expected in all cases, in which the method of Williams is strictly applicable, i. e. in which the processes with $\eta \gg 1$ do not play an essential rôle and the integrals of the type (13) converge rapidly.

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BIOPHYSICS

ON THE MECHANISM OF MUSCULAR ACTIVITY

By J. I. FRENKEL, Corresponding Member of the Academy

I. General Scheme

Recent progress in the physics and chemistry of polymeric substances, and rubber in particular, opens the way to a theory of the action of the most familiar but least understood polymeric substance—muscles.

The likeness between the elastic properties of rubber and of muscles in the relaxed state is a quite trivial fact. It should be mentioned that this likeness refers also to the order of magnitude of the elastic stress of a stretched piece of rubber or muscle (in the latter case the maximum stress amounts to 12 kg/cm^2) ([1]). Still more significant is the likeness between the thermoelastic properties of the two substances: a relaxed muscle, just as rubber, is warmed by adiabatical stretching instead of being cooled as ordinary bodies are([2]).

X-ray studies of the two substances have revealed a close similarity in their molecular structure; while rubber is built up of long hydrocarbon chains, myosin molecules are formed by long peptid chains, which in the equilibrium state of both substances are more or less irregularly curled. This curling, after Astbury([3]), is a characteristic feature of myosin, which distinguishes it from the keratin (both α and β) of hair and wool.

The elastic and thermoelastic properties of rubber are explained, according to Kuhn([4]) and Mark([5]), by the tendency of the long hydrocarbon chains it is built up of to curl in an irregular manner, which corresponds to the largest possible value of probability or entropy (and which is rendered possible by the free rotation of the elements of the carbon chain about the simple bonds connecting them). The elastic pull produced by stretched rubber is due to the kinetic energy of the thermal agitation of the elements of the hyrdocarbon chains which tend to assume the original more curled and consequently more probable configurations.

This point of view must obviously be applied to the case of a stretched muscle in the relaxed state too. At the same time it may serve as a basis for a correct interpretation of the mechanism of muscular activity, as manifested by the phenomenon of muscular contraction or more exactly muscular excitation*.

As has been already pointed out by Astbury([3]), if a relaxed muscle is comparable with natural rubber, an excited one should be compared to

* The term «contraction» is inadequate because an excited muscle cannot only contract but also be stretched by the application of an external force.

vulcanized rubber (ebonyte being the extreme case of the latter). I believe this to be more than a simple analogy, the process of muscular excitation exactly corresponding, in my opinion, to that of vulcanization in the case of rubber.

The latter process consists, as well known, in the bridging up of separate hydrocarbon chains by atoms of sulphur. A chain of total length l, linked in this way with other chains at n different (more or less equidistant) places, behaves so far as the elastic and thermal properties of the corresponding substance are concerned in the same way as n free chains of length l/n. Now, according to the theory of Kuhn and of Mark, the elasticity modulus of rubber is inversely proportional to the length of the molecules constituting it. This is why vulcanization results in a hardening of rubber, quite analogous to that of an excited muscle.

It seems thus quite probable that the mechanism of muscular excitation consists in the formation of lateral bonds between the curling myosin molecules by some substance which remains in a chemically bound state so long as the muscle is relaxed. The release of this substance is the immediate chemical cause of muscular excitation.

I am not quite certain as to the exact nature of this substance. It must be set free in the process of decomposition of phosphagen followed by the formation of lactic acid, whose oxidation gives the energy necessary for the regeneration of the phosphagen. It seems probable that this «vulcanizing» substance consists in the last issue simply of hydrogen ions. This hypothesis is supported hy the fact that when a muscle is excited by means of an electric current, the wave of contraction begins at the cathode.

Leaving the discussion of this question to chemists, I wish but to point out the important fact that under normal conditions this «vulcanizing» substance is very rapidly eliminated by the oxygen dissolved in the substance of the muscle or forwarded to it by the blood stream.

This elimination or «devulcanization» process constitutes the specific characteristics of a living muscle, distinguishing it from rubber or from a dead muscle (which becomes permanently rigid until it begins decomposing). The primary energy of a living muscle is the energy of this «devulcanization» or, in other words, of relaxation.

The current view that in the contracted state the muscle has more energy than in the relaxed one is quite erroneous. As a matter of fact the converse must be true (just as a demagnetized body has more energy than when it is magnetized). More exactly, we must speak not of energy, but of free energy. The additional elastic free energy of excited muscle corresponds not to an increase of potential or kinetic energy, but to a decrease of entropy at a given temperature. This increase of elastic free energy takes place at the expense of the chemical free energy, accumulated in the relaxed muscle and partly dissipated as heat during its contraction.

The mechanism of muscular activity, sketched above, may be effective in that case only if what is called the «normal length» of a muscle in the relaxed state actually corresponds to a certain amount of stretching above the equilibrium length. The elastic pull due to this «normal stretching» fails to be observed either because it is too small or (in the living organisms) because it is compensated by an equally small pull of the antagonistic muscles. The «vulcanization» of a muscle, underlying its excitation, produces a large increase of the pulling force for a given degree of the stretching tending to decrease the latter. This tendency would lack in the absence of an initial or «normal» stretching.

If a piece of rubber was stretched to a certain extent Δl and thereafter vulcanized «isometrically», that is without change of length, its pull could be greatly increased, remaining however proportional to the extension Δl (so long as the latter is not too large). Under «isotonic» vulcanization at a constant value of the pulling force the piece of rubber would contract in just the same way as a muscle does under ordinary working conditions. To make the rubber model of a working muscle more adequate, one should succeed in alternating the introduction of the vulcanizer with that of «devulcanizer» or «relaxer» in the form of some active substance (dissolved in a neutral filler, just as oxygen is dissolved in the 80% of muscular water) capable of removing the vulcanizing substance by chemically combining with it.

II. Special Points.

1. Energy Relations. The elastic free energy of a piece of rubber of unit volume with a normal length l_0 stretched to a length $l = l_0 + \Delta l$ is equal to $\frac{1}{2} E \frac{(\Delta l)^2}{l_0^2}$, where E is the elasticity modulus. According to the Kuhn-Mark theory, the latter is inversely proportional to the length of the hydrocarbon chains—so long as they are freely curling. In the presence of lateral links, numbering n per each molecule, the effective «free» length is reduced by $(n+1)$ times and the elasticity modulus, as well as the elastical free energy, is increased $(n+1)$ fold for the same value of the extension Δl.

As easily seen n may be identified with the number of vulcanizing atoms per one molecule of the natural rubber. Denoting with U the decrease of chemical energy corresponding to the binding of two chains by such an atom, we can represent the change of the total free energy of a piece of rubber consisting of N long-chained molecules due to isometric vulcanization in the form:

$$(\Delta F)_l = \left(\frac{1}{2} E_0 \frac{(\Delta l)^2}{l_0^2} - UN \right) n, \qquad (1)$$

where E_0 refers to the case $n = 0$ (natural rubber). If U is larger than $\frac{1}{2} E_0 \frac{(\Delta l)^2}{l_0^2 N}$, that is the elastic free energy per one molecule of the non-vulcanized rubber, its vulcanization will be accompanied by a decrease of free energy. This seems to be the case with isometrically contracting muscles, the energy balance being maintained by the evolving of an amount of heat equivalent to $-(\Delta F)_l$.

In the case of isotonic vulcanization, corresponding to a constant pulling force $f = E_0 (n+1) \frac{\Delta l}{l_0^2}$, we get in a similar way a change of free energy

$$(\Delta F)_f = -\left(\frac{1}{2} \frac{l_0^2 f^2}{E_0 (n+1)} + UN \right) n \qquad (2)$$

which is always negative.

2. Kinetics of the Vulcanization and Devulcanization Processes in a Muscle. The contraction of a muscle is ultimately due to the decomposition of some stored up substance of higher energy (say, phosphagen) which we shall denote with A, yielding the «vulcanizing» particles A' (hydrogen ions, according to our hypothesis). This reaction requires a certain activation energy. However, because of its exothermal character it cannot be limited to a definite point, but must be propagated to a surrounding region in a way similar to the propagation of flame. This accounts for the fact that muscular contraction started at a certain point by any kind of stimulus is propagated in the form of a contraction wave

along the whole muscle. The laws of this propagation are probably the same as those referring to the propagation of flame in a gas mixture after the reaction-chain theory of Semenov and his collaborators. The rôle of the oxidation process is here played by the vulcanization process, that is the binding up of A' with the myosin molecules to form lateral links between them.

The actual oxidation process which slightly lags behind that of vulcanization consists in the destruction of these links by oxidizing the particles A' constituting them. Whether this process, leading to the relaxation of the muscle, has a chain-like character and can be propagated in a way similar to the propagation of the vulcanization process, seems to be open to question.

Owing to its secondary character and relative slowness, it must become effective only after the contraction process is completed and proceeds accordingly in all points of the muscle (where there is a supply of oxygen) simultaneously just as if it were not capable of propagation*.

3. Connection between the Elastic Pull of a Muscle and the Velocity of Its Deformation. As has been shown by Fenn(1), the elastic force of an excited muscle is a function not only of its length but also of the rate of change of this length with the time $\frac{1}{l_0}\frac{dl}{dt}=v$. For a given value of l (and a constant excitation stimulus) it is larger when $v>0$ (extension) than in the opposite case. It seems that in both cases its dependence on the velocity v can be represented by the exponential formula: $f=f_0e^{av}$, where f_0 corresponds to $v=0$ (static force). This result is utterly inconsistent with Hill's view that the dependence of f upon the velocity is due to viscous friction. The latter must play but a secondary rôle which is the same for both signs of v.

The simplest interpretation of the above law follows from the fact that the extension of a muscle either relaxed or contracted is accompanied by heating, while a shortening—by cooling of the substance (just as in the case of rubber). Now because of the very large temperature coefficient of the vulcanization reaction a stimulus of the same intensity must produce a larger excitation effect when the temperature is raised (extension) than when it is lowered (shortening); if the reaction velocity is proportional to $e^{-\frac{w}{vT}}$ (w—activation energy), then a small change of temperature with v will result in an exponential dependence of f upon v (for a given l). In working out a quantitative theory of this effect one must take into account the heat conduction between the myosin substance and the inert filler, which probably reduces the rise or fall of $t°$ to an amount proportional to the velocity v.

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* If vulcanization be compared to combustion, oxygen in a muscle plays the rôle of water stopping the propagation of fire.

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CHEMISTRY

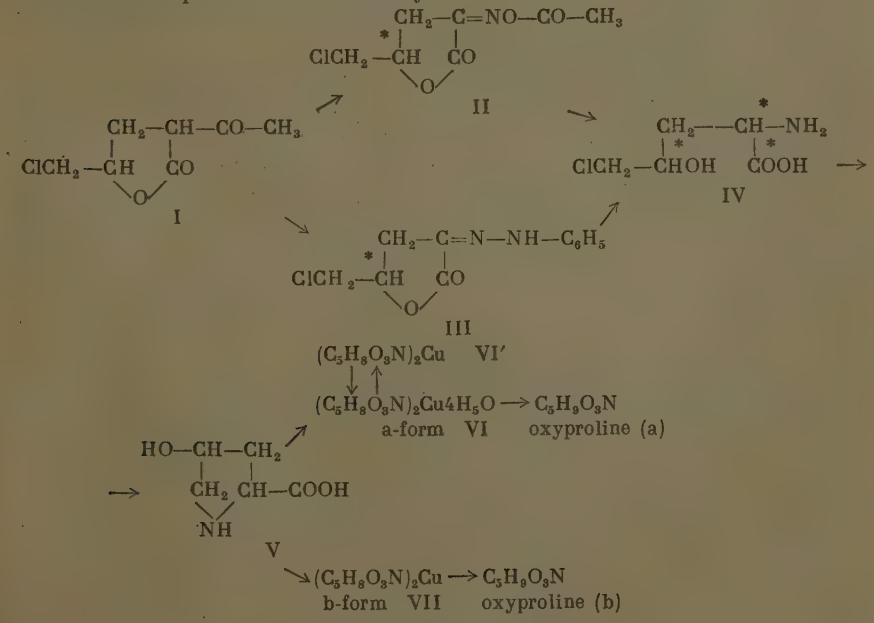
SYNTHESIS OF OXYPROLINE (γ-OXYPYRROLIDINE-α-CARBOXYLIC ACID)

By V. FEOFILAKTOV and A. ONIŠČENKO

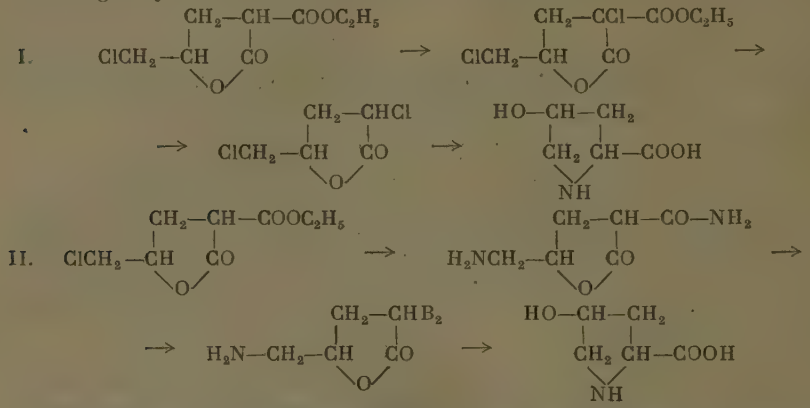
(Communicated by A. E. Favorsky, Member of the Academy, 29.IV.1938)

In our study of the behaviour of certain homologues of α-substituted butyrolactones, in particular of α-aceto-δ-chloro-valerolactone (I), when treated with nitrous acid and benzenediazonium, we stated, in complete accordance with our former observations, the easy formation of the corresponding oxime, its acetate (II) and phenylhydrazone (III), with the splitting off at the same time of the acetylgroup.

When we applied the method of reduction with tin and hydrochloric acid (which had justified itself in such cases) to the reduction of the oxime-acetate (II) and of the phenylhydrazone (III), we obtained in both cases, with a rather good yield, α-amino-γ-oxy-δ-chloro-valeric acid (IV). This acid, as it was to be expected from a 1.4-amino-haloid-derivative, showed the capacity to be converted under the influence of ammonium hydracide into oxyproline (V). This manner of obtaining oxyproline, by the comparative simplicity of the operations involved, can be of considerable interest as one of the methods for the synthesis of this aminoacid, which is extremely interesting from the viewpoint of biochemistry.



l-oxyproline was discovered for the first time by E. Fisher (1902) among the products of hydrolysis of gelatin and casein (1903). Later it was found by different investigators among the products of hydrolysis of different proteins; its betains were also discovered in certain plants. The synthesis of oxyproline was realized by H. Leuchs, Brewster[1] and W. Traube, Jehow, Tepohi[3], who proved definitely its configuration. This synthesis was realized in the following way:

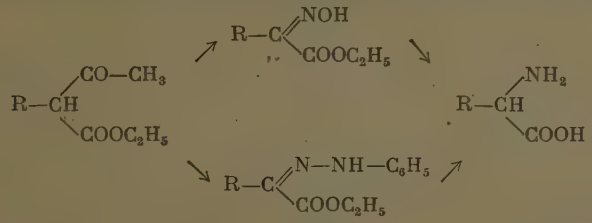


The dichloride, as well as oxyproline itself, was obtained by them as two racemates—the a- and b-forms. The a-form of oxyproline was resolved into its optically active components in the form of quinine salts. The *l*-isomer proved to be identical with the natural *l*-proline. The synthesis accomplished by us is the third synthesis of this compound. The initial α-aceto-chloro-valerolactone(I) was synthesized by us according to W. Traube and E. Lehmann[3]. The oxime-acetate (II) was obtained by shaking α-aceto-δ-chlorolactone (under cooling) with a water solution of sodium nitrite. After gradual addition of diluted sulphuric acid, the oxime-acetate immediately fell out as a paste of colourless crystals having after recrystallization out of alcohol or benzene a melting point 115—116°. Yield—81% of the theoretical amount. The formation under these conditions not of the free oxime (α-oxy-imino-δ-chloro-valerolactone), but of its acetate, points to an anomalous course of this reaction.

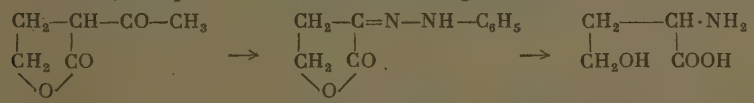
The phenylhydrazone of α-keto-δ-chloro-valerolactone (III) was obtained with a yield of 83% by gradually adding α-aceto-δ-chloro-valerolactone (I) to a cooled solution of phenyldiazonium chloride (1 : 1 mol.) with efficient stirring. After recrystallization from alcohol or acetic acid the product was obtained in needles of a pale yellow colour, having melting point 185 —186°. With concentrated sulphuric acid and a drop of a kalium bichromate or ferric chloride solution (Bülow's reaction) it shows an intensive red-violet coloration. The yield of α-amino-γ-oxy-δ-chloro-valeric acid (V) is 56%, when it is the oxime-acetate that is reduced. The phenylhydrazone gives a yield of 44%. In agreement with its configuration it can exist as a mixture of two racemates (a- and b-forms). The pure b-form obtained in a crystalline state has a melting point 165.5—166.5°. When treated with ammonium hydroxide it yields the Cu-salt of the b-form of oxyproline—octaedric crystals of a dark violet colour (VII). The a-form of the amino-acid (IV), which is more soluble, remains in the mother liquids. When the latter are treated with ammonium hydroxide, small crystals coloured pale-blue fall out. That is the Cu-salt of the a-form of oxyproline (VI) with 4 mols of crystalline water. This salt has the peculiar property of transforming itself into its anhydrous form (VI'), not

only when being dried or when standing in a dessicator, but even when a transparent water solution of the hydrated salt (VI) is boiled for a short time. The solution becomes turbid because of the precipitation of the anhydrous form (VI'). Its conversion back into the hydrated form can be realized either by dissolving it in an extremely large amount of boiling water (1 : 700), wherefrom the hydrated salt falls out after cooling, or by transferring it back into free oxyproline. The free oxyproline (a-form) was obtained by decomposing the Cu-salt of the a-form with hydrogen sulphide. Yield—about 88%. After rectystallization (out of water by precipitation with alcohol) it forms lustrous colourless crystals having the shape of rhombs and parallelograms. Melting point, when heated rashly, 255—256° with decomposition.

The comparatively simple synthesis of α-amino-γ-oxybutyric acid from α-aceto-butyrolactone, that was achieved by us formerly, and the synthesis of α-amino-γ-oxy-δ-chloro-valeric acid which we used for the synthesis of oxyproline and which is described in this paper, may be considered as a particular case of the synthesis of α-amino-acids from monosubstituted acetoacetic esters, taking place according to the following general scheme:



When R—is $HOCH_2—CH_3—$, i. e. when acetoacetic ester is condensed with ethylene oxide, the process takes the following course:



In the case of epichlorhydrine: $R = ClCH_2—CHOH—CH_2—$ the process leads to the formation of $ClCH_2—CHOH—CH_2—CH(NH)_2—COOH$.

Wishing to prove the generality of this method, we tried isobutyl-acetoacetic ester with the hope of obtaining leucine. Actually, we realized this transformation and obtained it very easily and with good yields. The work concerning the further use for the synthesis of α-amino-acids of this method, which is apparently general and which has many advantages over other methods, is being continued.

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CHEMISTRY

ON AZOXYCOMPOUNDS. THE MECHANISM OF THEIR FORMATION DURING THE REDUCTION OF NITROCOMPOUNDS WITH SODIUM AMALGAM

By V. O. LUKASHEVITCH

(Communicated by M. A. Iljinsky, Honorary Member of the Academy, 9.V.1938)

It may seem that the exhaustive explanation of the mechanism of formation of azoxycompounds has been given long ago: according to the usual conception, they should be formed through condensation of nitrosocompounds and β-arylhydroxylamines, produced during the reduction of nitro- and nitrosocompounds with sodium amalgam.

This classical explanation, common for all cases of reduction, is based on the following: nitrosocompounds and β-arylhydroxylamines, as it has been originally revealed by Bamberger(1), form swiftly and quantitatively the corresponding azoxycompounds.

On the other hand, β-arylhydroxylamines cannot themselves serve as a source of production of azoxycompounds, for phenylhydroxylamine, according to Bamberger(2), converts to azobenzene in the alcohol-alkali medium; moreover, Haber(3) has shown that its rate of decomposition is not high under these conditions.

It should be noticed that, according to such conception of the mechanism of formation of azoxycompounds, one is compelled to conclude presumptively that the rates of reduction of nitrosocompounds are much lower than the rates of their condensation with β-arylhydroxylamines.

When studying the reaction mixtures produced during the action of amalgam on the alcohol solutions of nitrocompounds in an atmosphere of nitrogen, the author detected arylhydroxylamines, whereby their amount proved to be abundant as soon as the reaction mixture was acidified directly after conclusion of interaction with amalgam. It was, however, sufficient to delay neutralization as, for instance, in the case of reduction of nitrobenzene at room temperature, even for ten minutes, to make arylhydroxylamine almost completely disappear.

This gave the author the following indications. In the first place, contrary to the above-mentioned presumptive statement, the rate of conversion of nitrosocompounds to β-arylhydroxylamines was much higher in our conditions than the rate of azocoupling. And, secondly, in contrast with the observations of Haber who examined the action of alcohol solution of alkali on phenylhydroxylamine in the absence of any other reactionable substances, we dealt with the process of rapid decomposition of β-arylhydroxylamines.

What are the possible reasons of this phenomenon?

It is well known that phenylhydroxylamine converts readily to azoxybenzene in alkaline medium in the presence of air[4]. It was natural to suppose that nitrocompounds may take the part of oxygen of the air. The author has ascertained, indeed, that β-arylhydroxylamines and nitrocompounds readily and almost quantitatively convert in alcohol-alkali medium to corresponding azoxycompounds upon removal of air.

In order to gain a proper notion of the rates of such conversions, the author submitted nitrocompounds to the action of small amounts of β-arylhydroxylamines for extremely short periods of time, and estimated then the remaining amounts of arylhydroxylamines.

Doing so he has found this process to proceed in general most rapidly. The rates of interaction of various nitrocompounds and arylhydroxylamines are arranged in approximately the same manner, as the rates of azoxycoupling of the latter with respective nitrocompounds[5].

We have especially to emphasize that treating a nitrocompound with a small amount of amalgam instead of the given arylhydroxylamine, and as far as possible keeping other conditions unaltered, we always found almost exactly the same amount of arylhydroxylamine formed (Table 2).

From all the stated above, the author came to the following interpretation of the mechanism of reduction of azoxycompounds. Nitrosocompounds reduce very rapidly when acted upon by the aforesaid agent, and convert therefore into β-arylhydroxylamines. The reducing agent introduced in portions is thus replaced by a new, less vigorous one, namely β-arylhydroxylamine, interacting in turn with nitrocompound and the generating nitrosocompound; the azoxycompounds are therefore produced according to the following equation:

$$3RNH.OH + RNO_2 \longrightarrow RN{=}\underset{\underset{O}{\|}}{N}R + 3H_2O.$$

It is difficuet to decide a priori, which explanation is more suitable in each particilar case of reduction, as one must take into consideration not only the nature of the reducing agent, but also temperature, concentration of alkali, rate of introduction of the given reducing agent, etc. Nevertheless, the author will here refer to some instances.

When nitrocompounds are reduced with stannous oxide in alkaline medium, the main part of azoxycompounds seem to form according to our scheme, since, according to Goldschmidt, the rates of reduction of nitrosocompounds should be infinitely high ([6]).

Such probably is also the origin of azoxycompounds in the case of electrolytic reduction in alkaline medium with low potentials on the cathode([7]), in which case the rates of reduction of nitrosocompounds ought to be extremely high too.

On the contrary, the classical interpretation alone is applicable to the cases of electrolytic reduction in an acid medium.

When the reduction on cathode in alkaline medium is going on under usual conditions, or, for instance, in the case of reduction with zinc dust in the presence of alkalies, both interpretations must probably be taken into account.

Experimental

Reduction of Nitrocompounds with β-arylhydroxylamines. 1. Azoxybenzene. Nitrobenzene (3 g) was dissolved in 6.6% alcohol solution (50 g) of sodium hydroxide; phenylhydroxylamine (4. 36 g), dissolved in alcohol (4 g) and sealed in a small bulb, was introduced into a glass vessel containing the above solution in an atmosphere of nitrogen. The bulb was broken by shaking the vessel vigorously upon obtaining high vacuum in the vessel and swiftly heating the contents

up to 55—60°. For the first few moments the mixture became of a black-greenish hue, but the colour lightened almost at once and turned yellow. In five minutes the vessel grew cool; the acidified reaction mixture was diluted with water and repeatedly extracted with ether. The ether solution was washed with alkali and then with acid of moderate concentration. After evaporation of ether and nitrobenzene, there was found azobenzene (5.24 g) having a freezing point 34.9°. No aniline was found.

2. Nitrobenzene (4 g) dissolved in benzene (40 ml), 25% aqueous solution of sodium hydroxide (45 g) and phenylhydroxylamine (2.18 g) were vigorously shaken in a glass vessel in an atmosphere of hydrogen for 15 minutes at 65—70°. After the benzene solution had been treated in the above manner, there were found 2.52 g of azoxybenzene (m. p. 34—35°) and 0.048 g of aniline. On a line with azoxycompounds some minute quantities of monoamines have been obtained from the interaction of *m*- and *p*-nitrotoluenes with due amounts of β-arylhydroxylamines under almost the same conditions.

o-azoxytoluene. *o*-nitrotoluene (3.3 g), ethyl alcohol (43 g), 50% aqueous solution of sodium hydroxide (6.7 g) and *o*-tolyl-hydroxylamine (4.92 g) in 5 ml of alcohol were taken. The mixture was heated for 25 minutes at 70°. There was found 6.0 g of *o*-azoxytoluene, m.p. 57—58.5°.

Reduction of Nitrocompounds with Sodium Amalgam. In Table 1 experiments on reduction of nitrocompounds at comparatively low temperatures are given. Into a glass vessel with 0.01 mol of nitrocompound dissolved in 25 g of alcohol and cooled to 10°, in an atmosphere of nitrogen, some amalgam was introduced in a small bulb, whereupon the vessel was vigorously shaken for a few seconds. Then a cooled to 0° solution of sulphuric acid (1 g H_2SO_4 in 40 ml of water) was added immediately to the reaction mixture; the latter, separated from mercury and bits of glass from the broken bulb, was diluted with cold water to 500 ml and then 15 ml of 15% silver nitrate was added to it.

After some hours the deposit of silver was filtered off, washed with water and alcohol and burnt. By the weight of silver the quantity of arylhydroxylamine was calculated relatively to the amount of sodium taken.

Table 1

Nitrocompound	Amount of 0.5% amalgam, g	Amount of silver found, g	Amount of β-arylhydroxylamine in % to the theory
1. Nitrobenzene	87.4	0.695	67.8
2. »	87.4	0.130	12.7
3. *o*-nitrotoluene	87.0	0.848	82.3
4. *m*- »	87.4	0.900	87.8
5. *m*- »	86.8	0.610	59.3
6. *p*-nitrotoluene	87.2	0.650	63.7
7. *m*-nitrochlorbenzene	88.0	0.042	4.1
8. *m*-dinitrobenzene	44.1	Non detected	—
9. Nitrobenzene	90.2	—	58.0
10. *m*-nitrotoluene	88.4	—	69.2

In exp. 2 the mixture was acidified 10 minutes after the interaction with amalgam, in exp. 5 to the alcohol solution of *m*-nitrotoluene 4 g of 32% aqueous solution of sodium hydroxide were added. Naturally enough, arylhydroxylamines were obtained in these experiments in lesser quantities. In exp. 9 and 10 arylhydroxylamines were isolated in free state, the acidified

reaction being diluted for this purpose with 50 ml water and upon adding sodium acetate treated as described before(8).

Table 2 makes it possible to compare the amount of β-arylhydroxylamines that can be detected while reducing nitrocompounds by means of sodium amalgam, with those remaining in case of interaction of definite amounts of arylhydroxylamine with nitrocompounds. Due care was taken to secure possibly equal conditions for the comparative experiments. The amount of nitrocompound had been in all cases 0.01 moles, that of amalgam or arylhydroxylamine 6—8 g and 0.040 g respectively; the amount of 2.5% alcohol solution of alkali was 20 g. The amalgam or arylhydroxylamine in alcohol solution (a few drops of alcohol) were introduced in a bulb into solution of nitrocompound in an atmosphere of nitrogen. Then the glass vessel was vigorously shaken, due attention being paid to make the period of interaction in all cases equal as far as possible. The reaction was broken off by means of a swift neutralization with 0.9—1.0 g of sulphuric acid dissolved in 25 ml of water and cooled to 0°; the subsequent treatment and the estimation of arylhydroxylamine were performed as it has been described above.

Table 2

Nitrocompound	Amalgam g	β-arylhydroxylamine, g	Duration of the reaction, sec.	Temperature, °C	Silver, g	β-arylhydroxylamine, %
1. Nitrobenzene	6.73	—	5	19	0.060	76.0
2. »	—	0.04	5	19	0.0576	72.7
3. »	6.87	—	5	50	0.046	57.0
4. »	—	0.04	5	50	0.0426	53.8
5. »	7.12	—	5	63	0.027	32.3
6. »	—	0.04	5	63	0.0618	88.1
7. *o*-nitrotoluene	6.67	—	5	19	0.069	88.4
8. »	—	0.04	5	19	0.0618	88.1
9. *m*-nitrotoluene	7.22	—	5	19	0.061	72.0
10. » »	—	0.04	5	19	9.052	74.1
11. » »	7.51	—	3	63	0.046	52.3
12. » »	8.05	—	5	63	0.029	30.7
13. » »	—	0.04	5	63	0.026	37.0
14. » »	—	0.100	60	57	0.012	6.9
15. *m*-nitrochlorbenzene	7.2	—	5	19	0.013	15.7
16. » »	—	0.041	5	19	0.010	18.2

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CHEMISTRY

SOLUBILITY OF SODIUM BORATES IN SATURATED SOLUTION OF NaCL IN THE PRESENCE OF MAGNESIUM SULPHATE

By N. A. SCHLESINGER, I. B. FEIGELSON and A. I. SPIRIAGINA

(Communicated by N. S. Kurnakov, Member of the Academy, 10.V.1938)

With the discovery in 1934 of the Inder deposit of borates in Western Kazakhstan there came to the fore a number of problems referring to the domain of applied science as well as pure geo- and physico-chemistry.

Notwithstanding the great variety of borates occurring in the Inder deposits, as well as their relatively great quantity, this deposit alone, though having true economic significance, cannot meet all the requirements in boron on the part of the national economy of the USSR, so great is the need in respect of this element finding itself a vast range of application in various branches of industry, medicine and agriculture.

Accordingly, the question very naturally arises of searching for new deposits. The solution of this problem would seem to lie along two possible paths: one of them is the direct investigation of brines in various salt-water bodies, while the other one consists in a laboratory study of a number of equlibrium systems, without which it is impossible to establish the genesis of borates.

Insofar as the majority of the boron bearing brines of salt-lakes studied by us come close to sea-water(1) in composition, it would seem but quite natural to begin the solution of our problem from the system Mg, Na‖Cl, SO_4, as the one which approaches most closely to natural conditions.

The classical investigations of vant-Hoff do not give a full picture of the very complex physico-chemical processes taking place in nature in course of the formation of borates; wherefore it follows that further elaborate studies are needed, all the more since this problem appears to be one of the most difficult in the given field.

The problem which we propose to consider consists in determining the solubility of sodium tetraborate along the line Na_2 Cl_2—$MgSO_4$ of the isotherm of the marine system from point XIII to XIV(2) with temperature +35 and +50°, in the presence of various quantities of $MgSO_4$, and within the ranges of the concentration of this compound as occurring in the brines of salt-lakes. The temperatures selected by us, viz. +35 and +50°, permit, first of all, of studying the system under conditions approaching the natural ones ($t°$= +35°) and, secondly, they make it possible to study the solubility of both hydrated forms of $Na_2B_4O_7$, since at a temperature of +35.5° in solutions saturated with NaCl [as this was established by vant-Hoff's(3) investigations] $Na_2B_4O_7 \cdot 10H_2O$ passes into $Na_2B_4O_7 \cdot 5H_2O$.

Isotherm +35° (Table 1). With given temperature, $Na_2B_4O_7$ contains 10 molecules of water. Its solubility at + 35°(4) is equal to 5%, i. e. 4.57 molecules $Na_2B_4O_7$ fall to the share of 1000 molecules of water.

In saturated solutions of sodium chloride $Na_2B_4O_7 \cdot 10H_2O$ dissolves in quantities much lesser than 1.60 molecules, which constitutes 35.03% of the solubility of this compound in water. The quantity of water falling to the share of 1 molecule $Na_2B_4O_7$ in aqueous solutions is equal to 218.8 molecules and in solutions saturated with NaCl—to 625.0 molecules.

Table 1

Isotherm+35°

Solution №	Per cent by weight					Molec. salts per 1000 molec. water			
	Spec. gravity	$MgSO_4$	NaCl	$Na_2B_4O_7$	Sum of salts	$MgSO_4$	Na_2Cl_2	$Na_2B_4O_7$	Sum of molecules
	—	—	—	5.00	5.00	—	—	4.57	4.57
1	1.2034	—	25.93	1.42	27.35	—	54.96	1.60	55.56
2	1.2199	1.34	25.05	2.14	28.53	2.79	53.97	2.66	59.42
3	1.2373	2.64	24.10	3.10	29.84	5.61	52.89	3.93	62.43
4	1.2512	3.82	23.17	3.76	30.75	8.22	51.52	4.83	64.57
5	1.2670	5.03	22.14	4.68	31.85	10.99	50.03	6.11	67.13
6	1.2834	6.00	20.92	5.83	32.85	13.29	47.91	7.71	68.91

In solutions saturated with NaCl and containing various quantities of magnesium sulphate, the solubility of $Na_2B_4O_7 \cdot 10H_2O$ is going up and when the composition of the solution is such as given under № 4 (8.22 molecules of $MgSO_4$, 51.52 molecules of Na_2Cl_2) surpasses the solubility of $Na_2B_4O_7 \cdot 10H_2O$ in water, reaching 4.83 molecules per 1000 molecules H_2O. Further increase in the concentration of $MgSO_4$ in solution contributes still more to the solubility of $Na_2B_4O_7 \cdot 10H_2O$, reaching 7.71 molecules (solution № 6).

Thus, we are able to draw the conclusion that in solutions saturated with ordinary salt too, the presence of magnesium sulphate, with $t° = +35°$, increases the solubility of $Na_2B_4O_7 \cdot 10H_2O$.

Isotherm + 50° (Table 2). With +50° we meet with another hydrated form—$Na_2B_4O_7 \cdot 5H_2O$, and here again we are able to draw the conclusion that, just as with $t° = +35°$, the solubility of $Na_2B_4O_7 \cdot 5H_2O$ in saturated solutions of NaCl in the presence of $MgSO_4$ tends to grow with increased concentration.

Table 2

Isotherm+50°

Solution №	Per cent by weight				Per 1000 molecules water			
	$MgSO_4$	NaCl	$Na_2B_4O_7$	Sum of salts	$MgSO_4$	Na_2Cl_2	$Na_2B_4O_7$	Sum of molecules
1	1.51	24.38	3.75	29.64	3.19	53.36	4.74	61.29
2	4.15	22.27	4.32	30.74	8.92	49.52	5.55	63.99
3	7.02	20.65	4.69	32.36	15.46	47.01	6.17	68.64

This phenomenon should thus be considered as a common one for both hydrated forms of $Na_2B_4O_7$.

It is necessary to note that equilibrium with $t° = +50°$ is, in a strange way, established much later than with $t° = +35°$; in all probability the influence

of $Na_2B_4O_7 \cdot 5H_2O$ makes itself manifest here. It is not possible to compare the solubility of $Na_2B_4O_7 \cdot 5H_2O$ in saturated solutions of NaCl in the presence of $MgSO_4$ (as found by us) with the solubility of the same compound in pure water, since in aqueous solutions, with $t° = +50$, $Na_2B_4O_7 \cdot 5H_2O$ passes into $Na_2B_4O_7 \cdot 10H_2O$.

The data given for isotherms +35 and +50° have definitely shown us that the solubility of both hydrated forms of $Na_2B_4O_7$ in saturated NaCl solutions in the presence of $MgSO_4$ tends to increase. Wishing to study this phenomenon with greater precision, we undertook a number of experiments on the solubility of $Na_2B_4O_7 \cdot 10H_2O$ at the same temperature of +35°, but in the presence of $MgSO_4 \cdot 7H_2O$ alone. We found the following:

1) The concentration of $Na_2B_4O_7 \cdot 10H_2O$ in solutions, which are far from being saturated with $MgSO_4$, is less than the solubility of $Na_2B_4O_7 \cdot 10H_2O$ in water (4.57 molecules in water and 4.05 molecules in the solutions in question in the presence of 2.35 molecules of $MgSO_4$).

2) With growth in concentration of $MgSO_4$ the solubility of $Na_2B_4O_7 \cdot 10H_2O$ falls in the absence of NaCl.

Conclusions

1. An investigation has been made of the solubility of $Na_2B_4O_7 \cdot 10H_2O$ with $t° = +35°$, as well as of the solubility of $Na_2B_4O_7 \cdot 5H_2O$ with $t° = +50°$ in solutions saturated with NaCl and in the presence of increasing quantities of $MgSO_4$.

2. The solubility of $Na_2B_4O_7 \cdot 10H_2O$ in saturated NaCl solutions is markedly lowered.

3. In solutions saturated with NaCl in the presence of varying amounts of $MgSO_4$ the solubility of $Na_2B_4O_7 \cdot 10H_2O$ rises to a magnitude, surpassing the solubility of this compound in pure water.

4. The solubility of $Na_2B_4O_7 \cdot 10H_2O$ in aqueous solutions of $MgSO_4$ is less than in pure water, while further increase of the quantity of $MgSO_4$ still more reduces the solubility of $Na_2B_4O_7 \cdot 10H_2O$.

5. The investigation carried out by us leads us to suppose that under natural conditions the borates of sodium are capable of being leached out by solutions containing both ordinary salt and magnesium sulphate. In course of a further mixture of the water stream with solutions containing mainly NaCl and poor in $MgSO_4$, the dissolved borate ought to fall out of solution.

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CHEMISTRY

EQUILIBRIUM AND KINETICS OF THE FORMATION OF OXONIUM COMPOUNDS

By A. T. GLADYSHEV and J. K. SYRKIN

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1. The interaction of two complex molecules uniting into one may be hindered if the probability of the transition state is but small. In this case the velocity is many times less than that expected according to the simple collision theory. This may take place both in the case of solutions and in reactions between gases, independently of whether the process proceeds homogeneously in the volume or heterogeneously on the walls. The existence of minor steric factors characteristic of the formation of quaternary ammonium salts in solutions was shown by us for the case when the same reaction was proceeding in the gaseous phase(1). In this connection the kinetics of the formation of oxonium compounds from its gaseous components is of interest. It is well known that gaseous dimethyl ether reacting with hydrogen chloride gives $(CH_3)_2$ OHCl. It is easy to follow the course of this reaction by observing the change in pressure. However, the study of the kinetics of the reaction is impossible owing to the fact that when both of the gases are admitted into the reaction vessel pressure drops immediately and equilibrium sets in. Wegscheider(2) has measured the degree of dissociation of $(CH_3)_2$ OHCl at different temperatures. He found that below 10°C, owing to adsorption on the walls of the vessel, the equilibrium constant fails to be stable. According to the data of this author, the heat of dissociation lies in the neighbourhood of 5000 cal. Maas and Morrison(3) have studied equilibrium in the temperature range of 0—29° C. The constants obtained at different temperatures appear to be of different value. This may be seen from the fact that the heat of reaction fluctuates within a rather wide range. The mean value of energy lies in the neighbourhood of 5500 cal. In the work of Iitaro Shidei(4) the equilibrium constant of this reaction was measured at temperatures varying from 1 to 19° C. The thermal effect calculated from the equilibrium constants is equal to 6725 calories.

The method we used consisted in the following: a definite amount of each substance was admitted into the reaction vessel and the equilibrium constants were then determined by pressure drop. Next, the mixture was heated to a certain other temperature, whereupon pressure was measured again. Such heating was effected until full dissociation set in and, in course of further temperature rise, pressure increased according to the equation holding for perfect gases without dissociation. Thus, it was possible to effect a number of measurements with each gas mixture within a considerable temperature

range. It should be borne in mind that at low temperatures a part of the substance is adsorbed on the walls, whereby a certain error is introduced into the determination of gas composition. The fact that adsorption may markedly distort the results may, in particular, be seen from the fact that if 200 mm dimethyl ether be introduced into the reaction vessel, pressure is apt to fall by something like 10 mm.

In Table 1 are to be found temperature, general pressure, equilibrium pressure of each substance in mm mercury, and finally equilibrium constant with pressure in atm.

Table 1

T	P	P_{HCl}	$P_{(CH_3)_2O}$	P_{ox}	K
260	96.2	47.5	43.7	3	1.05
274	469.5	208.5	231.0	30	0.4734
283	493.7	224.1	247.4	22.2	0.3043
293	518.7	239.5	263.7	15.5	0.1862
303	493.6	221.9	261.7	10	0.1309

If we are to ascribe the same weight to experiments conducted at different temperatures and if we carry out calculations according to the method of least squares, then for the equilibrium constant we obtain:

$$K = 10^{-6.237} e^{\frac{7400}{RT}},$$

which gives $\Delta H = -7400$; $\Delta S = -28.5$ and $\Delta F_{25} = 1093$.

If, for computation, we take the data obtained at higher temperatures (293 and 303°) when adsorption is less, we obtain $\Delta H = -6250$ cal. Both our own data and the evidence obtained by other authors show then that the heat of formation of $(CH_3)_2OHCl$ lies in the neighbourhood of 7000 cal. The reaction proceeds very rapidly and equilibrium sets in practically at the very moment when the gases mix. This points to the fact that the activation energy is very small. The supposition of Latimer and Rodebush([5]) who postulated in this case the presence of a hydrogen bond seems to be very likely. If the reaction did not consist in the combination of acid hydrogen with ether oxygen by means of a weak side-valency or through dipole forces, and if the reaction were connected with a marked change in structure and with a loosening of the HCl molecule, then, probably, the process would be connected with an appreciable activation energy. We may add to this that measurements of the Raman spectrum of $(CH_3)_2OHCl$, at not too low temperatures, show that there is no essential difference between the spectrum of the oxonium compounds and the spectra of ether and acid([6]). It is interesting to compare these data on the energy of the hydrogen bond with the results obtained from other sources. The decomposition of the double molecules of formic acid calls for 14 125 cal. ([7]). As it is necessary here to break two hydrogen bonds, this gives 7060 cal. for the energy of a single bond. The energy of dissociation for the double molecules of acetic acid calculated from the vapour pressure measurements at different temperatures equals 14 900 cal., according to Fenton and Garner([8]), and 16 400 cal., according to the recent work of McDugall([9]). This gives 7450 and 8200 cal. respectively for the energy of the hydrogen bond.

A comparison of these results shows that in all cases, when there is an interaction between bound oxygen of the ether or carbonyl group and the hydrogen of another molecule, the thermal effect approaches 7000 cal. In

this connection the following is worth noting. In alcohols association is mainly determined by the hydrogen bond. If we are to divide the energy of van-der-Waals's interaction into the dispersion, induction and orientation parts, then it is reasonable to suppose that the energy of the hydrogen bond is close to the sum of the latter two parts. In the cases of methyl and ethyl alcohol 6500 cal.([10]) fall to the share of Keesom and Debye forces.

2. An attempt was made to investigate the kinetics of the reaction between dimethyl ether and hydrogen bromide. These two substances are known to give an oxonium compound. Experience shows that if HBr and $(CH_3)_2O$ are mixed at the temperature of liquid air, a crystalline white mass is formed which melts depending on the composition at —18—9° and boils at 3 to 5°. The reaction proceeds in time with a measurable velocity. Experiments were conducted at three temperatures: 1°, 10° and 20°. At the moment of gas admission pressure was rapidly going down. 186.7 mm of each substance was taken; at the moment of mixture, total pressure was equal to 308.3 mm instead of 373.4 mm, i. e. pressure fell at once by 65.1 mm. Thereupon it continued to fall very slowly. In about nine hours pressure fell additionally by 60—65 mm.

In Table 2 are to be found the results of experiments at 1° and 20°. The first column shows time in seconds, while the second one indicates general pressure. Both substances were originally taken in equal quantities.

Table 2

Temperature 1°C. Initial pressure 373.4 mm		Temperature 20°C. Initial pressure 379 mm	
Sec.	General pressure	Sec.	General pressure
0	308.3	0	345.1
9 780	275.0	9 720	332.7
24 540	250.4	26 700	313.8
32 580	240.4	34 860	303.5
40 020	231.5	40 920	295.7
49 800	222.8	50 400	283.8
66 420	210.8	65 640	269.0
76 440	205.6	78 300	260.5
84 660	201.8	86 520	255.2
—	—	91 920	251.7

The velocity of the reaction depends on the order in which the reactants are admitted into the reaction vessel. If HBr is admitted first, the reaction proceeds at a rate indicated in Table 2. But if ether is the first to be admitted, the reaction will be considerably slowed down. If 187.7 mm of every substance is admitted, pressure at the moment of admission falls by 19—20 mm. The further course of the reaction is equally slowed down. Pressure falls by about 60 mm in approximately 19 hours.

The phenomenon is apparently connected with the adsorption on the walls of the vessel. If ether is the first to reach the fresh surface, its molecules are oriented by oxygen towards the surface. This causes steric difficulties for the reaction, since it is difficult for the HBr molecule to come close to oxygen and enter into reaction, insofar as the outwardly directed CH_3 groups are a handicap for the reaction. On the other hand, if HBr is first introduced into the vessel, it occupies space on the surface and the oncoming molecules of ether can react more easily. Special experiments were undertaken in order to ascertain the adsorption of ether and HBr. If a definite volume of ether

corresponding to a pressure of 200 mm is introduced into the vessel, pressure falls by 10—15 mm on admission. Analogous experiments with HBr show that adsorption is then less and that pressure falls only by 4—5 mm. From these data it is possible to draw the conclusion that the adsorbed layer is polymolecular. A liquid film is formed on the surface, in which the reaction takes place.

Comparison of the course of the reaction at 1° and 20° shows that velocity slows down with increase in temperature. At 20° at the moment of gas admission pressure falls by 33.9 mm, instead of 65 mm at 1°. The further course of the reaction is also slowed down. This effect is due to the decrease in adsorption with increasing temperature.

Calculation of absolute velocity gives the following: in a vessel with a volume of 500 cm^3, during about 80 000 sec., pressure falls by 100 mm. This corresponds to the formation of $2.2 \cdot 10^{16}$ molecules of oxonium compound per second. The surface of the vessel is equal to 350 cm^2. Thus, $6.1 \cdot 10^{13}$ molecules are formed per unit surface per second; in other words, a monomolecular layer of oxonium bromide is formed on the average during 15—20 sec. The number of double collisions per unit surface calculated in the usual way is approximately equal to 10^{20}. The apparent energy of activation is not great. Its exact determination is difficult because the kinetics are very complicated and not conforming to one and the same equation at 1° and 20°.

There is no doubt that other reactions too proceed with the formation of oxonium bromide which makes the kinetic interpretation much more difficult.

It can be seen from our experiments that oxonium chloride is formed immediately after the mixing of the gases. In the case of bromide the reaction is measurable in time. Thus, a more polar hydrogen chloride reacts with a smaller activation energy. Judging by the velocities at 1° and 20°, which differ not much, the true activation energy stands close to the heat of adsorption. The reaction is heterogeneous. It is easy to see that with an energy of activation of 12 000 cal., which is the estimate of the heat of adsorption, it would have proceeded with a measurable rather high velocity in volume. Hence it may be concluded that the reaction of oxonium bromide formation has a very low steric factor.

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BIOCHEMISTRY

HYDROXYLAMINE FORMED IN PLANTS IN THE COURSE OF NITRATE AND NITRITE ASSIMILATION

By D. M. MICHLIN

(Communicated by A. N. Bach, Member of the Academy, 6.V.1938)

A hypothesis advanced by Meyer and Schultze(1) as early as by the end of the last century supposed aminoacids to form in plants not only through amination of organic acids and their derivatives, but also by way of intermediate formation of oximes. Since then one more than once had recourse to this hypothesis because of its being chemically well grounded. Some experimental evidence in its favour is to be found in a paper by A. N. Bach(2).

The reduction of nitrates to ammonia goes over a series of intermediate products, the hydroxylamine being one of that number. It is distinguished by its intense chemical activity, especially by its entering readily into combination with the carbonyl group to form oxime even in strongly diluted solutions.

The formation of formaldehyde in the process of photosynthesis(3), as well as the experimentally proved formation of acetaldehyde in plants(4), afford the possibility for oximes to be synthesized at the expense of the hydroxylamine formed in the plant in the course of nitrate reduction.

Ketoacids are also capable of producing aminoacids through formation of respective oximes and their further reduction. But in spite of having all chemical grounds to admit the hypothesis of aminoacid formation through oximes, one was long unsuccessful to prove the presence of the latter, nor that of hydroxylamine, in plants. At the same time it has been established by a series of physiological experiments that ammonia is formed in the plants growing in nitrate or nitrite mediums(5). However, we may as yet not be confident of ammonia forming directly from nitrate or nitrite. Its formation may possibly be a result of disamination of aminoacids. As to hydroxylamine, attempts have been made to isolate it from the plants in the form of a cupric compound combined with dimethylglyoxime, according to Čugaev(6). This method, however, is lacking sufficient sensitivity. It was only in 1926 that another method build up by Blom(7) afforded the possibility to reveal the presence of hydroxylamine in concentration of $1 : 10^7$. This method is based on the hydroxylamine being oxidized, under strictly controlled conditions, by iodine to nitrous acid and the latter estimated according to Griess. It has been worked out in detail by Lemoigne (8) who established the formation of hydroxylamine in *Sterigmatocystis nigra* and some higher plants.

Strictly following the method of Blom-Lemoigne, I found up to 0.12 mg of hydroxylamine per 1 g cells in *Chlorella*. However, in none of the higher plants enumerated in Table 1 did I succeed to find preformed hydro-

Table 1

Formation of Hydroxylamine in Leaves and Roots of Plants Following Infiltration of Solution Containing 0.06% Potassium Nitrate, 0.04% Formaldehyde and 0.05% Ascorbic Acid (in terms of ml hydroxylamine per 1 kg of fresh vegetable material)

Data	Plant	Composition of mixture	Hydroxylamine
15. I. 37	Hortensia leaves	Nitrite + formaldehyde	1.06
	Do	Do	0.98
5. IV. 37	Potato sprouts	Nitrite + formaldehyde	2.12
		Nitrite + formaldehyde + ascorbic acid	3.88
		Water	0
21. IV. 37	Potato sprouts	Nitrite + formaldehyde + ascorbic acid	4.02
	Do	Do	3.24
	Do	Water	0
2. VII. 37	Potato leaves	Nitrite + formaldehyde	4.10
		Nitrite + ascorbic acid	6.25
		Water	0
17. VIII. 37	Potato leaves	Nitrite + formaldehyde + ascorbic acid	6.12
		Nitrite	0.03
		Formaldehyde	0.08
		Water	0.08
13. II. 38	Sprouts 7 days old	Nitrite + formaldehyde	2.72
		Nitrite + formaldehyde + ascorbic acid	4.06
23. II. 38	Sprouts 8 days old	Nitrite + formaldehyde + ascorbic acid	5.25
		Nitrite + ascorbic acid	0
15. III. 38	Sprouts 9 days old	Nitrite + formaldehyde	5.15
		Nitrite + formaldehyde + ascorbic acid	8.76

xylamine. Yet, if the leaves or potato sprouts are immersed in a solution of 0.04% formaldehyde and 0.06% KNO_3 and so kept for 48 hours in a room at 18—20° C, the formation of hydroxylamine may be established. The yield of hydroxylamine is doubled, if a solution containing formaldehyde and nitrite is infiltered in the leaves or sprouts. Ascorbic acid promotes the yield of hydroxylamine which in some experiments amounted to 8 mg per 1 kg of fresh leaves. Besides, there may always be observed a slight increase of ammonia content compared to that in control leaves infiltrated with pure water. The surplus of ammonia on the average does not exceed the amount of hydroxylamine formed. The formation of hydroxylamine may not be put down to the action of microorganisms, the latter being possible only in the presence of formaldehyde. Though ascorbic acid be present the nitrite alone without formaldehyde is not apt to form hydroxylamine in the leaves.

It is thus formaldehyde which under our experimental conditions combines hydroxylamine in the form of oximes. Under natural conditions oximes may be formed at the expense of other aldehydes or even ketones.

Experimental

100 g of leaves or potato sprouts, not above 1/2 mm thick, were put into an exsiccator from which the air had been drawn out by means of an oil-pump. Then 400 ml of a solution containing 0.06% potassium nitrite, 0.04% formaldehyde and 0.05% ascorbic acid were thrown through a tap on the leaves or sprouts respectively. The estimation of nitrite showed its content in the solution to decrease by 8—12% immediately upon infiltration. No estimation of formaldehyde contained in the liquid after infiltration was done. As to ascorbic acid, a paper by Rubin and associates(9) has shown that it penetrates into the leaf at vacuum infiltration.

Table 2

Formation of Ammonia in Potato Leaves Infiltrated with a Solution Containing 0.06% of Potassium Nitrite, 0.05% of Formaldehyde and 0.05% of Ascorbic Acid (control leaves, infiltrated with water, in terms of ml ammonia per 1 kg of fresh leaves)

		mg NH_3
1	Experiment	18.8
	Control	16.4
2	Experiment	13.20
	Control	10.16
3	Experiment	13.14
	Control	9.24
4	Experiment	16.88
	Control	14.20

The vegetable material immersed in the solution was allowed to stand for 48 hours in a thermostat at 30° C. It was then ground in the same liquid after up to 10% of icy acetic acid had been added to the latter. To break up the nitrite 15 g of urea were added to the mixture. In order to remove nitrogen oxides, which might possibly occur in the mixture, the latter was boiled for 5 minutes. Thereby an almost complete precipitation of proteins was achieved. The mixture was allowed to stand for 24 hours at room temperature. After filtration and treatment with a small amount of animal charcoal, the liquid obtained by second filtration was colourless and did not contain any oxides of nitrogen*. Tests for nitrous and hyponitrous acid resulted in negative reaction.

Hydroxylamine was estimated in 10 cm^3 of colourless transparent liquid according to Blom-Lemoigne. For the sake of comparison there was always prepared a solution of hydroxylamine sulphate containing 0.01 ml of solution. Not in a single case did the control solution, devoid of vegetable material, give Blom reaction. To estimate ammonia, 2 ml of colourless solution neutralized with respect to litmus were taken. After distillation in Parnas apparatus the ammonia was estimated by means of a scale photometer.

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* Charcoal adsorbs 30—40% of hydroxylamine.

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BIOCHEMISTRY

THE STRUCTURE OF PROTEIN MOLECULES AND THEIR CATALYTIC PROPERTIES

By D. L. Talmud, Corresponding Member of the Academy

1. The structure of a protein molecule is no usual problem of chemistry, not only because proteins are the most complicated highmolecular substances, the structure of which is far from being clear. The biochemical rôle of proteins makes the question of the structure of the protein molecule one of the most important questions of all the natural sciences. The solution of this question would open the way to the investigation of the metabolism in living organisms, as a series of elementary chemical processes, securing the continuous reproduction and accumulation of definite proteins.

The hypothesis of the structure of protein molecules recently advanced by Wrinch(1) seems to offer such a possibility. However, this hypothesis is still merely the result of a mathematical analysis of the problem. It requires therefore first to be experimentally checked. Once experimentally verified, this hypothesis will become a theory, on the basis of which it will be possible to give a new interpretation to the biochemical rôle of protein. The experiments described below are the first attempts made in this direction.

2. At the present time three states of proteins are known: 1) the «fibrous» state, characterized by a regular structure of the molecular chain and by the crystalline structure of the «fibre»; 2) the amorphous state, characterized by an irregular interlacement of «curled» molecular chains, and 3) the «globular» state of protein molecules in solution, which crystallize with a large quantity of water from a mother liquid of definite composition. It is very probable that «globular» proteins are nearest to the active biochemical state; the amorphous state is connected with the denaturation of «globular» proteins, while the «fibrous» state is obtained by the extension of an amorphous protein.

It is therefore natural that the greatest interest lies in the structure of the molecule of «globular» protein, although the latter is the least studied. It is, however, established beyond doubt that the «globular» proteins are no mixtures, but homogeneous substances of a definite molecular weight (about 35 000 or a multiple of this value)(2). For a series of proteins with the molecular weight about 35 000 Bergmann and Niemann(3) have shown very convincingly that the molecule consists of 288 amino-acid residues.

The mean features of Wrinch's hypothesis are: 1) the assumption that all amino-acid residues form a «cyclol» structure of di- and triazine rings alternating in a certain way and 2) the assumption that the «cyclol» plane can be folded into polyhedrons, the surface of which consists only of a certain number of amino-acid residues. For the protein with a molecular weight of about 35 000 exactly 288 residues are obtained.

The experimental check of Wrinch's hypothesis was carried out by us in the following way.

As may be seen from Fig. 1, representing the scheme of the cyclol structure, the lacunae in the faces of the «globular» molecule form a space enclosed by 12 hexagons. The side groups of the α-amino-acids, however, constrict these lacunae, making their area of the order of the benzene molecules (only in the case when at least a part of the amino-acids consists of glykokoll, the lacuna will have a larger size).

This consideration has lead us to the idea that molecules with a cross-section smaller than the area of a benzene molecule must diffuse through the lacunae inside the protein molecules, evidently filled with the solvent. If thereafter an «intraglobular» reaction is produced, followed by an increase of the molecular size up to or above that of the benzene nucleus, then the molecules of the reaction products will be unable to penetrate into the inter-

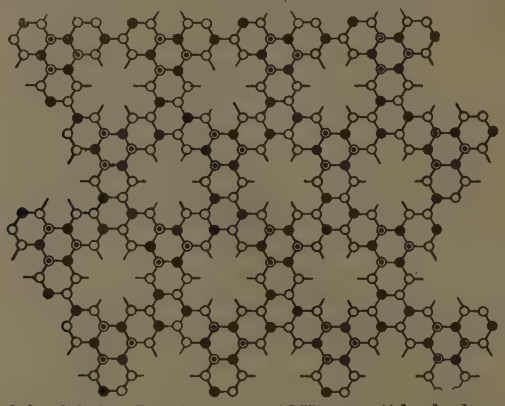

Fig. 1. The «cyclol» fabric. ● = N; ○ = C(OH), peptide hydroxyl, upwards; ⊙ = C(OH), peptide hydroxyl downwards; ○—=CHR, direction of side chain initially outwards; ○-=CHR, direction of side chain initially upwards. The median plane of the lamina is the plane of the paper. The lamina has its «front» surface above and its «back» surface below the paper (from Wrinch's paper).

globular space. This reaction must proceed at a temperature causing no protein denaturation.

For carrying out such an «intraglobular» reaction we have chosen the ethyl ether of glykokoll possessing, according to Fischer(4), the remarkable property of transforming into 2-5-dioxo-piperazine at room temperature in a water solution.

$$\begin{array}{l} C_2H_5OOC.CH_2NH_2 \qquad CO\text{——}CH_2 \\ \qquad\qquad\qquad\qquad \longrightarrow NH \qquad\quad NH \quad +2C_2H_5OH. \\ H_2N.CH_2COOH_5C_2 \qquad CH_2\text{——}CO \end{array}$$

To exemplify the above we refer here to one of the typical experiments: to a water solution of a twice precipitated crystalline egg albumin, cooled to 0° C and neutralized by ammonia solution, a water solution of glykokoll ethyl ether was added. The total volume of the mixture amounted to 50 cm³ and contained 3.3282 g protein and 5.6375 g glykokkol ethyl ether. The mixture was next heated for 24 hours at 35° C and left for 48 hours more at room temperature remaining all the time absolutely transparent. But already in a few hours after the beginning of the experiment a thin coarse-crystalline precipitation was built up, which proved to consist of dioxo-piperazine

(0.3374 g). This precipitation being removed, no more dioxo-piperazine deposited from the mixture could be observed as late as 5 days after the beginning of the experiment.

The solubility of dioxo-piperazine in 50 cm^3 of protein solution of the same concentration and composition was determined besides in a separate experiment; 0.6522 g of dioxo-piperazine proved to be transferred into the solution.

We established in this way that from the dioxo-piperazine, which was formed in the «intraglobular» reaction, 0.3374 g+0.6522 g=0.9896 g was formed between the «globules» (it was formed at the expense of 1.7882 g of ethyl ether of glykokoll). Hence it follows that 5.6375 g—1.7882 g=3.8493 g of ethyl ether of glykokkoll, from which 2.296 g dioxo-piperazine was formed, remained inside the «globules».

Special measurements of the kinetics of the formation of dioxo-piperazine from a water solution of glykokoll ethyl ether have shown that practically all the ether was transformed into dioxo-piperazine during the experiment with protein (Fig. 2). Since this reaction is of the second order ($k_2 = 12.1 \cdot 10^{-3}$), the dependence of its velocity on the concentration of ether is very great, while the measurement of the kinetics was carried out at a concentration (3%, temperature 35° C), which was three times smaller than in the experiment with protein.

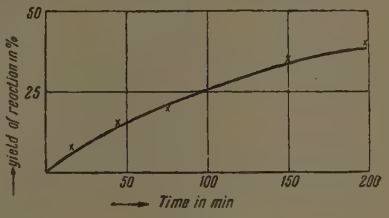

Fig. 2.

We have separately measured the adsorption of glykokoll ethyl ether on egg albumin, denaturated by heating. The adsorption took place from 4% solution at 0°C when the velocity of transformations of ether into dioxo-piperazine could be neglected. The adsorption proved to be equal to 0.18 g per 1 g of coagulated protein. This experiment shows that adsorption gives a six times smaller amount of ether held up by protein compared to the «intraglobular» reaction.

No less coclusive evidence in favour of the «intraglobular» reaction and against the adsorption is obtained when comparing the following data:

Molecular weight of egg albumin [2]	34 500
Specific weight of egg albumin	1.3
Quantity of egg albumin taken for the experiment	3.1382 g
Volume of protein taken for the experiment	2.41 cm^3
Number of protein molecules in the quantity taken for the experiment	$5.5 \cdot 10^{19}$
Surface of one «globule» of egg albumin	5914.5 $Å^2$
Volume of one «globule» of egg albumin	43 214 $Å^3$

Even if one assumes that each hexagon in the «cyclol» structure can hold up 2 molecules of glykokoll ethyl ether, or that $^1/_3$ of all the surface of the «globule» is covered with molecules of glykokoll ether (cross-section—20.4 $Å^2$), then we obtain 70 molecules of glykokoll ether per «globule», the total quantity of protein used in the experiment being capable of taking up $5.3 \cdot 10^{21}$ molecules of glykokoll ether. From the data given below it is easy to calculate the quantity of dioxo-piperazine which must be found inside the protein «globules» after the «intraglobular» reaction.

Specific weight of glykokoll ethyl ether	1.0275
Volume of 1 molecule of glykokoll ether	165.5 $Å^3$
Number of molecules of glykokoll ether which can be taken up by 1 «globule» of egg albumin	261
Specific weight of dioxo-piperazine	1.083
Volume of [1]) molecule of dioxo-piperazine	173.7 $Å^3$
Number of molecules of dioxo-piperazine which can be taken up by 1 «globule» of egg albumin	248

The experiment has shown that one «globule» contained 200 molecules of dioxo-piperazine, formed from 400 molecules of glykokoll ether, while the total number of glykokoll ether molecules transformed by the «intraglobular» reaction into dioxo-piperazine amounted to $22.4 \cdot 10^{21}$. This is 4.2 times the quantity of glykokoll ether which could be adsorbed on the surface of all the «globules».

At the same time the number of dioxo-piperazine molecules found within the «globules» is very near to the maximum of molecules which could be taken up by all the «globules»:

Number of dioxo-piperazine molecules in all the «globules» (found experimentally) . . . . . $11.2 \cdot 10^{21}$
Maximum number of molecules of dioxo-piperazine which could be taken up by all the «globules» . . . . . $13.6 \cdot 10^{21}$

3. The adsorption undoubtedly precedes the «intraglobular» reaction. It is the only to explain the catalytic course of the «intraglobular» reaction, whilst in the interglobular medium the velocity of the reaction is considerably smaller. The catalyzer is here the «globular» molecule of protein itself. In fact, if the reaction velocity of dioxo-piperazine formation inside the «globules» and between them would be the same, then a fraction only of the total amount of dioxo-piperazine could be found inside the «globules», which is equal to the fraction of the total volume of the solution occupied by the protein. In our experiment the volume occupied by the protein molecules (2.41 cm^3) is equal only to 0.05 of the total volume of the solution (50 cm^3). Therefore in the absence of catalytic action of the protein «globules», only 0.05 of all the dioxo-piperazine formed could be found inside the «globules». As a matter of fact about 0.68 of the total amount of dioxo-piperazine was trapped inside the «globules».

The catalytic character of «intraglobular» reaction may be explained in the following way. The molecules of glykokoll ethyl ether diffusing inside the «globules» are adsorbed on its inner surface (being probably linked mith enol OH-groups). The «cyclol» structure of the «globule» surface plays here the rôle of a «stamp» with respect to the adsorbed molecules of glykokoll ethyl ether, the distribution of which on the surface of the«stamp» reproduces exactly its geometrical structure. This advantageous preliminary distribution of the reacting molecules undoubtedly favours sterically the acceleration of the reaction. In contradistinction to the molecules of glykokoll ether the «stamped» dioxo-piperazine molecules do not possess adsorbable groups and accordingly are immediately desorbed, leaving their place on the «stamp» surface free for new molecules of glykokoll ether.

4. The experiments described above, confirming the hypothesis of the structure of protein molecules proposed by Wrinch, at the same time prove the existence of «intraglobular» reactions and the catalytic rôle of the protein structure with respect to these reactions. They open, moreover, the way for the hypothesis (perhaps yet poorly grounded) of the biocatalytic action of the protein in the accumulation and reproduction processes of a certain kind of protein. The molecule of the «globular» protein should be compared to an elementary cell just as a chain-molecule of cellulose, keratine and so on may be called an elementary fibre. It is easy in such an elementary cell to realize synthetic «intracellular» processes even when a dissimilation process takes place outside them. It is known that the derivatives of dioxo-piperazine are split up by some proteolytical ferments. This splittting up cannot, however, take place inside the elementary cell, since the ferment molecules are too large to penetrate there. The catalytic action of the structure of the protein molecule itself will contribute inside the «globules» to the synthesis of «cyclols» or their fragments from separate amino-acids, which are formed

in the process of «extracellular» dissimilation. The «intraglobular» synthetic processes are perhaps stimulated by the prostetic group, entering into the «cyclol» structure of protein or linked in some other way with its inner surface. At any rate, in the inside of an elementary cell a new protein molecule is synthesized, which gradually dehydrates the old molecular shell. When this dehydration causes denaturation, the old shell must decyclize and sets free the new protein molecule. It is of course thinkable that inside the large «globules» with a very large molecular weight a few molecules with a smaller molecular weight can be formed, and so on. This hypothesis may be called the hypothesis of the evolution of protein molecules.

Conclusions

1. The hypothesis of the structure of «globular» proteins proposed by Wrinch has been checked experimentally.
2. The existence of «intraglobular» reactions has been shown.
3. The catalytic character of «intraglobular» reactions has been proved and the molecular mechanism of the catalytic action of the protein structure discussed.
4. The hypothesis is advanced of a biocatalytical evolution of protein molecules.

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BIOCHIMIE

LES OXYDATIONS DANS LE VIN

Par S. M. MANSKAYA

(Présenté par A. N. Bach, de l'Académie, le 22.V.1938)

La notion actuelle de l'influence que l'oxygène exerce sur le vin manque de clarté car son action est en même temps nuisible et favorable à la qualité du vin.

Deux opinions prédominent dans la littérature oenologique. Une a été exprimée par Berthelot(1), qui professait la nuisibilité de l'oxygène, surtout pour les vins vieux. Il affirmait que le vin exposé au contact de l'air perd son bouquet et aquiert une saveur «éventée» désagréable.

L'autre opinion, avancée par Pasteur(2), proclama l'importance de l'oxygène dont la présence fut considérée nécessaire au développement normal d'un vin viellissant. L'expérience bien connue de Pasteur sur la conservation du vin en tubes scellés a démontré qu'un vin jeune, récemment fermenté, guardé à l'abri de l'oxygène (en tube pleins), avait préservé toutes ses qualités du vin jeune, tandis que le même vin conservé en contact avec l'oxygène (en tube incomplètement remplis) avait acquis les qualités d'un vin vieux—par la perte de l'odeur de cru, formation du bouquet, apparition des teintes dorées et brunes dans sa couleur.

Il n'y a pas longtemps qu'un livre a paru qui attira l'attention des oenologues—notamment celui de Ribéreau-Gayon(3) sous titre: «Contribution à l'étude des oxydations et réductions dans les vins». Ce livre contient un sommaire bibliographique des ouvrages sur les oxydations dans le vin, soumet à une analyse critique les notions actuelles de l'influence que l'oxygène exerce sur les qualités du vin, et décrit les résultats des travaux effectués par l'auteur lui-même. Ribéreau-Gayon a développé une méthode spéciale servant à déterminer la quantité de l'oxygène dissous dans le vin, la vitesse de sa combinaison et l'action des catalyseurs. Il attribue le rôle de catalyseurs oxydants aux traces des sels de fer et de cuivre, que l'on trouve dans tous les vins, et puis encore aux certaines substances organiques qu'il réunit avec des substances inorganiques sous un nom général d'«oxydants intermédiaires». Par ce nom l'auteur veut indiquer que ces substances sont aptes à être oxydées directement par l'oxygène moléculaire pour former des oxydants instables. En conséquence l'oxydation ayant lieu dans un vin comporte deux temps:

1) L'oxygène se fixe sur les oxydants intermédiaires.

2) Ces derniers transmettent l'oxygène actif aux éléments oxydables du vin.

En examinant les substances qui sont sujets à l'oxydation dans le vin Ribéreau-Gayon a trouvé que l'oxydation du tanin et des matières colorantes joue un rôle essentiel dans le développement des vins rouges.

Par son analyse du vieillissement des vins il a démontré que la participation de l'oxygène est nécessaire dans la première phase de ce processus. Cette phase terminée, le rôle de l'oxygène est assez secondaire et même peut être nuisible à la qualité du vin.

Les observations de Ribéreau-Gayon sur l'action des oxydants intermédiaires se rangent parfaitement dans le schéma d'oxydation développé par A. N. Bach.

Suivant les travaux de Manchot(6) et Warburg(7) l'action catalytique du fer sur l'autooxydation consiste en ce que $Fe^{\cdot\cdot}$ absorbe une molécule d'oxygène pour former le peroxyde FeO_2, et transmettre ensuite l'oxygène actif aux éléments oxydables. Cependant les expériences de A. N. Bach(4) ont mis en évidence que le fer intervient comme catalyseur pour accélérer l'action du peroxyde d'hydrogène qui est créé le premier. Si les conditions ne favorisent pas la formation du peroxyde d'hydrogène, la catalyse de l'autooxydation par le fer n'a pas lieu. Probablement les «oxydants intermédiaires» de Ribéreau-Gayon agissent en ensemble—catalyseurs inorganiques + peroxydes.

Le but de nos études était à déterminer la nature biochimique des processus responsables pour le développement du vin. Pour base nous avons pris les propositions suivantes.

Le vin est une solution contenant un ensemble de ferments dont l'activité varie selon l'âge du vin et la technique de son traitement. Les oxydations dans le vin sont de nature fermentative et se produisent suivant la théorie de A. N. Bach(8).

Les ferments oxydants qui contribuent à l'oxydation des éléments propres du vin consistent, suivant la terminologie de Bach, de l'oxygénase et de la peroxydase; l'oxydation comporte deux périodes:

1. Formation des peroxydes organiques (oxygénase + oxygène moléculaire).

2. Oxydation par ces peroxydes des éléments oxydables, ce processus étant activé par le ferment peroxydase.

Un schéma analogue des oxydations est présenté par la fermentation du thé.

L'auteur de cette Note(5) observa une absorption intense de l'oxygène par les feuilles du thé quand celles-ci étaient fraîches ou desséchées ou bien pendant qu'elles subissaient la torsion. Mais la quantité de l'oxygène absorbé au cours de la torsion ne se trouvait pas en équilibre avec le dégagement du gaz carbonique. Cela nous permet de conclure que certaines substances enrichies en oxygène s'accumulent dans les feuilles.

C'est au dépens de l'oxygène accumulé pendant la torsion que s'oxyde le thé au cours de sa fermentation.

La fermentation aboutit à lui donner la qualité d'infusion et le parfum que nous apprécions dans un thé prêt à consommation.

D'ailleurs, il a été démontré que l'oxydation qui a lieu au cours de la fermentation se développe aux dépens des peroxydes organiques formés pendant la torsion des feuilles. Le méchanisme de cette oxydation est tel que les peroxydes sont formés d'abord, au dépens de l'oxygène libre de l'air, et ensuite le tanin du thé est oxydé par l'hydrogène combiné du peroxyde qui est activé par le ferment peroxydase.

Les oxydations qui ont lieu dans un thé subissant la fermentation sont analogues à celles qui se passent pendant le développement d'un vin.

Ceci dit, nous avons admis que le développement du vin, au cours duquel ce dernier acquiert la saveur, couleur et odeur d'un vin vieux, se passe sous

l'oxydation des éléments propres par l'oxygène combiné (en forme de peroxyde) de l'air. La formation de ces peroxydes a lieu pendant l'aération maximum du vin jeune—garde en cave, mise en fûts et en bouteilles.

L'aération d'un vin vieux n'est point nécessaire pour la formation des peroxydes; l'oxygène est en excès et amène une altération des qualités précieuses déjà acquises par le vin. Nous croyons cependant possible que l'aération faible et lente par les bouchons des bouteilles et les bondes des fûts produit un effet favorable en redonnant de l'oxygène aux peroxydes, qui subissent une réduction lente.

Dans un vin fait l'activité des ferments est minime et le vieillissement s'écoule très lentement, durant des années.

En nous appuyant sur des considérations théoriques nous nous sommes proposé d'accélérer le vieillissement du vin au moyen des ferments oxydants. Nous avons fait des essais de laboratoire avec de différents échantillons (vins de liqueur, vins ordinaires rouges et blancs). Dans ces expériences nous avons étudié, par rapport à un temoin conservé à l'abri de l'air, l'altération des qualités du vin due à l'action des doses minimes du ferment peroxydase dissous dans l'eau. Outre la peroxydase, nous avons dans certain nombre de cas additionné le vin de peroxyde d'hydrogène.

Les essais de laboratoire ont établi des différences entre les témoins et les vins traités (en faveur de ces derniers) au double point de vue de l'analyse chimique et de l'estimation organoleptique. Nous avons eu recours à la méthode organoleptique vu que le vin est en général apprécié non par ses indices chimiques mais par sa saveur, sa couleur, son parfum.

La base vinicole de Moscou était le lieu de nos expériences sur l'oxydation biochimique du vin. Les échantillons suivants ont été mis à l'épreuve: vin ordinaire blanc № 22 (récolte 1936), Riceling № 61 (récolte 1933), Cabernet № 40 (récolte 1933), Bordeaux № 43 (récolte 1933), Madère № 31 (récolte 1933), Porto № 29 (récolte 1936), Porto № 30 (récolte 1937).

L'expérience comportait trois séries:

1) vin témoin,
2) vin additionné de ferment,
3) vin additionné de ferment et de peroxydase d'hydrogène.

Tous les échantillons ont été gardés en bouteilles pleines, bouchées.

La première dégustation a été faite deux jours, la seconde six jour après la mise en bouteilles.

Les dégustations ont démontré que par rapport aux témoins les vins additionnés de ferment avaient subi des modifications essentielles quant à leurs qualités principales—limpidité, couleur, bouquet et saveur. Dans tous les sept échantillons de la 2-e et 3-e série la couleur et surtout la saveur et le parfum avaient acquis le caractère d'un vin vieux. Le goût de cru propre aux vins jeunes avait disparu. Leur robe était devenue moins vive, les teintes brunes avaient apparu. Le vin avait une odeur agréable et un goût bouqueté.

Les vins ordinaires blancs et rouges additionnés de ferment avaient dévancé les témoins d'un point par rapport à la qualité et de deux ans par rapport au développement. La différence était particulièrement grande dans le cas des échantillons Porto № 29 (récolte 1936) et № 30 (récolte 1937) additionnés de peroxydase. Ces échantillons excellaient les autres par leur harmonie, bouquet et saveur, et l'on marqua leur qualité d'un et demi point plus par rapport aux témoins, tandis que leur développement était avancé de deux ans.

Il convient ici de noter que les vins ordinaires de la troisième série (additionnés de peroxydase et de peroxyde d'hydrogène) avaient «le goût de souris». Jusqu'à présent c'étaient les bactéries que l'on considérait résponsables pour ce goût dans le vin. L'apparition de cette maladie au cours d'une oxydation fermentative permet de l'étudier en détail au point de vue biochimique.

Donc, nous avons réussi à mettre en évidence la nature biochimique des oxydations dans les vins et de reproduire en laboratoire un développement accéléré moyennant la peroxydase.

Le développement du vin dans des conditions naturelles est un processus excessivement lent, il exige des années.

Le développement accéléré nous donne le moyen d'achever une étude des processus biochimiques dans un vin pendant quelques jours au lieu de la poursuivre plusieurs années. On pourra d'ailleurs commencer et achever l'expérience sur le même échantillon ce qui est très important, car l'étude d'un vin qui se développe durant des années est bien difficile à cause des modifications qu'il subit chaque saison suivant son traitement, conditions climatiques et conservation.

Plusieurs auteurs ont plus d'une fois essayé de vieillir le vin en faisant usage de différents accélérateurs comme l'ozone, le peroxyde d'hydrogène, le traitement chimique. Mais toutes ces méthodes ont produit un vin déprécié, éventé.

Or, la méthode biochimique d'accélérer le développement du vin nous donne, en dehors d'une accélération notable, un produit où les qualités précieuses d'un vieux vin sont préservées et même concentrées.

L'étude ultérieure doit concerner les modifications chimiques qui commandent le développement du vin.

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LITHOLOGY

ON THE «STRUCTURE» OF PORE SPACES OF ARENACEOUS OIL COLLECTORS

By P. P. AVDUSIN and M. A. ZVETKOVA

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Recently a number of articles appeared in the oil literature throwing light on the problem of the hydrodynamic processes of the fluid in a porous medium. These are before all the works of American scientists H. Herold, L. C. Uren, Ch. Fettke, R. D. Wiskoff, H. Botset, C. M. Newin, N. Hotting, Nutting, M. Muskat, etc.

Many of these investigators suggest very interesting and clever methods of determination of the rate of fluid filtration in a porous medium, and give formulae for the determination of zones drained by a drill-hole; however, the character of the processes of oil movement in the rocks remained obscure up to lately. This may be put down to the fact that, on the one hand, the principal value in these calculations has been the porosity coefficient, and the interpretation of the experimental data has been made assuming the existence of (theoretically possible) ideal types of pore channel structures (Lilley, Slikhter, Muskat and oth.), sometimes widely different from the actual structures of collectors; on the other hand, many sides of the complex of physico-chemical processes, taking place between the fluid and the polymineral solid phase of the collectors, have not been considered when computing the movement of oil in the rocks.

Yet, after some years ago accurate methods of micropetrographic analysis had been adopted for investigation of physical properties of collectors, the conditions of oil movement in a porous medium soon became clear. The investigators succeeded in presenting the actual structure of the pore channels with a complete distinctness and clearness; they showed the mineral composition of the grain and cement of the rock and approached the elucidation of the complex processes of oil movement in the enclosing rocks.

The first studies of the petrographic properties of arenaceous collectors in the USSR were started in 1929 in the Azerbaidzhan Scientific Research Institute for Petroleum (P. Avdusin). The experiments carried out in the Petrographic Laboratory of this Institute showed that sands of different petrographic composition with the same porosity differently respond to the process of oil filtration. In 1932 a note by V. P. Baturin appeared in the journal «Azerbaidzhanskoye Neftyanoye Khozyaistvo», where the author pointed to the influence of the mineralogical composition of rocks on the output of oil.

During 1934—1935 experiments were carried on in the Petrographic Laboratory of the Institute of Mineral Fuels (Academy of Sciences of the USSR) to determine the influence of the mineralogical composition of sands on the filtration of oil through them. A year later a systematic and thorough study was started of the petrographic properties of oil collectors viewed with regard to filtration of fluid hydrocarbons through them. The very first investigations towards this end showed that one of the most important factors, determining the character of the movement of fluid in the rock, is the «shape» of pore channels—the structure of the pore spaces.

In order to study the structure of the pore space of collectors a new method has been developed. It essentially consisted in that the samples under investigation were being saturated in a vacuum by a coloured but optically isotropic liquid, rapidly freezing in the cold; further very thin plane-parallel preparations were made of such a sample normally to the direction of the fluid flow, and these have been examined under the microscope sometimes using considerable magnification (up to 700). Bakelite or Canada balsam dyed with fuchsine or gentian violet have been usually employed in these experiments as a medium filling the pore spaces (method of N. M. Samsonova).

This method enabled the authors to start a systematic analysis of the structures of the pore spaces of arenaceous rocks. First the structures of artificial uneven-grained sands of different grades have been studied. The next cycle of research work consisted in a study of the structures of the most typical arenaceous rocks of the Transcaucasus oil deposits: the sands of the productive beds of the Apsheron peninsula, the Alyatsk district and the Prikurin region, as well as the sands and sandstones of the Mirzaan oil-bearing series (southern Kakhetia) have been studied.

Samples of rocks taken out both of natural exposures and drill-holes (column drills or «Craelius drilling») have been subjected to investigation; in both cases precautions were taken not to disturb the original structure of the rock.

An analysis of the material studied shows the processes of fluid transfer in a porous rock to be virtually determined by the so-called wet perimeter of the pore channel cross-section; P. Avdusin has suggested that the h y d r a u l i c c h a r a c t e r i s t i c s o f t h e p o r o u s s p a c e s t r u c t u r e s h o u l d b e p r e s e n t e d b y a r a t i o (or its reciprocal) o f t h e p e r i m e t e r o f t h e e f f e c t i v e c r o s s - s e c t i o n o f t h e f l u i d f l o w t o t h e c r o s s - s e c t i o n p e r i m e t e r o f t h e e q u i v a l e n t c y l i n d r i c a l p o r e c h a n n e l (ideal pore).

Ler L be the perimeter of the equivalent cylindrical pore channel section, and $\Sigma\ l=l_1+l_2+l_3+\ldots l_n$ — the sum of the perimeters of pore channel sections; then the characteristics of the pore space structure (denoted by Φ) may be expressed as

$$\Phi=\frac{L}{\Sigma l}. \qquad (1)$$

In order to calculate the value (of the coefficient) Φ the plane-parallel preparation of the analyzed rock has been examined under the microscope. With the aid of Abbe apparatus the most typical portion of the preparation within an area described by an arbitrary radius r has been drawn on paper.

Within the area of this circle with a radius R the effective porosity Π_{eff} is determined by integration, and the total length of the perimeters of the pore channel sections is calculated by means of a curvimeter.

Knowing the porosity Π_{eff} and the area of the drawing with a radius, the value R may be expressed by L and Π_{eff}.

In fact, if the porosity of the rock is Π_{eff}, the total area of the pore channel section within the area of the radius R is expressed by

$$d = \frac{\pi R^2 \Pi_{eff}}{100}. \qquad (a)$$

On the other hand, the ideal pore channel section, equivalent as to its area to the sum of the sections of all the pores within the area ΠR^2, is

$$d = \pi r^2, \qquad (b)$$

where r denotes the radius of the ideal pore channel which may be determined directly from

$$r = \sqrt{\frac{d}{\pi}}. \qquad (c)$$

The perimeter of the ideal pore channel is

$$L = 2\pi r. \qquad (d)$$

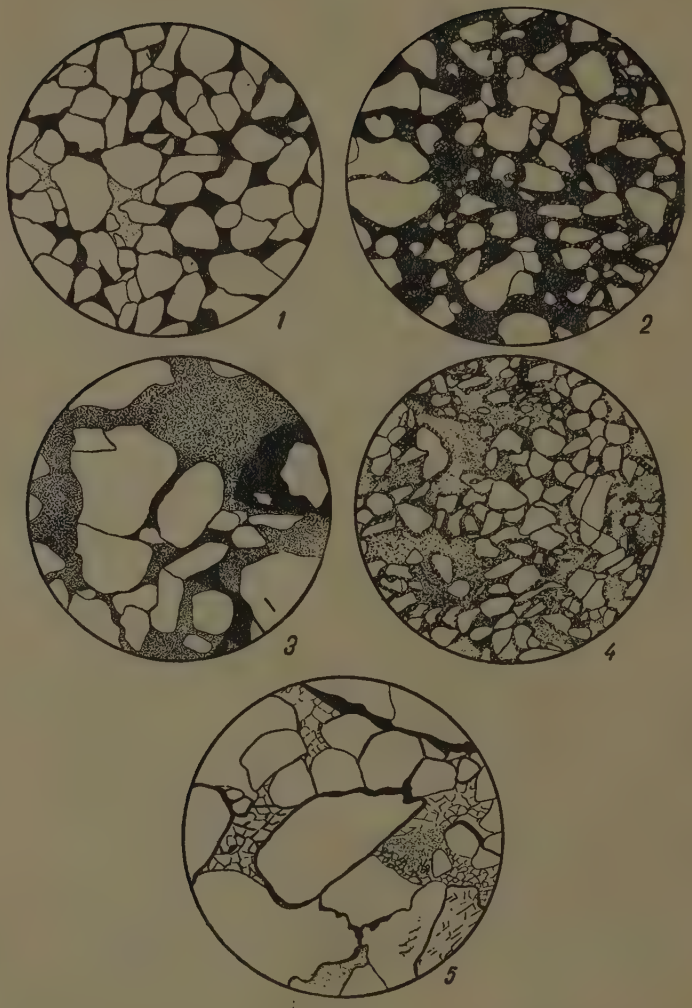
By placing in (d) successively (c), (b) and (a) and replacing L by the expressions obtained, the formula (1) may be presented as follows:

$$\Phi = 0.2\frac{\pi R\sqrt{\Pi_{eff}}}{\Sigma l} = 0.2\,\pi\,\frac{R\sqrt{\Pi_{eff}}}{l_1 + l_2 + l_3 + \ldots\, l_n}, \qquad (2)$$

where R is the radius of the area under investigation on the drawing of the plane-parallel preparation (in mm); Π_{eff}, the effective porosity of the rock area

	Porosity П*			Φ
	After Melcher	After Preobrazhensky	Effective Π_{eff}	
1. Medium-grained sand from the Ukhtinsk deposit	25.97	24.07	23.19	0.11
2. Medium-grained sand of the Balakh. series (the Apsheron peninsula)	33.53	33.84	32.36	0.12
3. Medium-grained sandstone from the middle division of the productive beds (the Apsheron peninsula, Azerbaidzhan)	22.70	22.20	20.40	0.07
4. Medium-grained sandstone from the productive beds (Airan-Tekyai, the Alyatsk district, Eastern Azerbaidzhan)	31.41	27.65 25.66 30.72	17.28	0.075
5. Medium-grained bituminous sandstone of the Mirzaan series (Southern Kakhetia) . .	16.03	16.30	13.48	0.047

* The porosity coefficients tabulated here have been obtained by the method of Melcher and Preobrazhensky, while the effective porosity Π_{eff} has been determined by integration (push-integrator of Glagolev, the Institute of Economic Mineralogy, integrator of Shand and others) in plane-parallel preparations by calculating the areas of pore channel sections.



Structure of pore spaces of arenaceous collectors of some oildeposits of the Union (black spots show the pore space, minute points—argillaceous cement).

Fig. 1. Medium grained quartz sand, Ukhta district, 3rd field; Devon, Chibuis series, III stratum (type A, porosity coefficient 23.19, coeff. Φ=0.10).

Fig. 2. Medium grained quartz sand, Apsheron peninsula; Neogen, productive soil, Sabunchinskaya series (type A—B, porosity coeff. 32.36, coeff. Φ=0.12).

Fig. 3. Coarse grained quartz sandstone (cement—calcium carbonate), Apsheron peninsula; Neogen, productive soil, middle division (type E).

Fig. 4. Medium grained polymict sandstone (cement—argillaceous substance and calcite), Airantek-yan, Eeastern Azerbaidzhan; Neogen, productive soil (type C, porosity coeff. 17.28, coeff. Φ=0.07).

Fig. 5. Coarse grained polymict sandstone (cement—calcite and argillaceous substance, Mirzaan ravine, Southern Kakhetia; Neogen, Shirak series (type D, porosity coeff. 13.48, coeff. Φ=0.06).

under investigation, determined by integration; $l_1+l_2+l_3+\ldots l_n$, the perimeters of the pore channel sections, determined in mm by means of a curvimeter.

The coefficient Φ, characterizing the structure of the pore space, is a very important value for hydraulic computations. Considered alongside of the effective porosity Π_{eff}, determining the volume of the pore channels draining the rock, it enables us to make out the character of the fluid movement in the porous collectors, and thus to solve the question as to the most efficient method of working the given oil deposit.

To exemplify the above, we will dwell upon a comparison between the pore space structures of several samples from arenaceous collectors of the largest oil deposits in the Soviet Union.

Microscopically these samples much resemble each other; they are all characterized by a high porosity and are therefore rich in oil. However, the structure of the pore spaces of these rocks from the largest oil deposits of the Soviet Union is altogether different and requires careful selection of the most expedient methods of working for collectors of different types.

The experiments made point to the presence of several types of collectors among the arenaceous rock of our deposits, differing from each other by the structure of the pore spaces. The most typical of the rocks analyzed are shown in Fig. 1, 2, 3, 4 and 5. These collector types conventionally denoted by *A*, *B*, *C*, *D*, etc., according to the combination of the values Π_{eff}, the medium grain size and the mineralogical composition of the rocks, may be in turn subdivided into classes, groups, etc.

A methodical analysis of the collectors may reveal the difference in their physical properties within the given deposit or even bed, and thus contribute to a more accurate estimation of the economic value of the deposit.

The principle of classifying oil deposits advanced by us which is based on the petrographic properties of the collectors (according to their hydraulic peculiarities) is important not only for the practical estimation of oil deposits, but may also be of certain scientific interest in solving regional problems of migration of fluid hydrocarbon compounds in the course of formation of oil deposits.

We have but taken the first step in the study of the petrographic properties of oil-bearing rocks. Further progress in this line needs more analytical data. After having gathered them systematically and inquired into the peculiar structure of collectors from individual oil-deposits, we shall have an abundant material for the second part of the work—the classification of oil-deposits (beds) of the Soviet Union according to their economic indices.

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GENETICS

ABNORMAL MEIOTIC PROCESSES INDUCED BY ACENAPHTHENE

By DONTCHO KOSTOFF

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Biologically active substance acenaphthene which was discovered by Shmuck(1) to induce morphological peculiarities in the way colchicine does has been used in the experiments here reported.

Sublimated particles from acenaphthene crystals induce various morphological changes in the plant organs (1,2) and irregularities in the mitotic processes, especially in cereals, but most frequently they induce chromosome duplications like colchichine solutions. The actions of the factors that condition the formation of a regular spindle and chromosome arrangements in a regular equatorial plate are inhibited. The chromosome divide but do not separate, thus polyploid nuclei are formed (1). In certain cases, when a separation begins but is not completed, binucleate and further polynucleate cells originate. This phenomenon also leads to polyploidy.

Acenaphthene particles interfere with the meiotic process too. Moist reagent tubes (glass) were covered from inside with acenaphthene crystals. Shoots with flower buds of *Nicotiana* species were covered with such tubes and closed from downside by cotton. Crystals were also brought on floral buds before covering them with tubes. Thus, floral buds were exposed to the action of the sublimating particles of acenaphthene in the closed tubes. After two days the procedure of the meiosis was studied in the floral buds treated with acenaphthene. Branches with floral buds from the same plants served for control.

Treated floral buds of *Nicotiana rustica* var. *humilis* ($2n=48$), *N. Langsdorffii* ($2n=18$), *N. longiflora* ($2n=20$), *N. glauca* ($2n=24$), *N. Cavanillesii* ($2n=24$) and *N. Sanderae* ($2n=18$) had numerous irregularities, some of which will be noted here.

During the meiosis no regular spindles were formed and the chromosomes failed to be regularly arranged in equatorial plates. Bivalent chromosomes were not polarly arranged, as they appear during the normal metaphases in untreated material. Their position remind of that occupied by them during diakinesis. They were only somewhat closely situated into one group. Bivalents gradually separate but at the same time they get gradually spread into the cytoplasm in small groups (*N. longiflora*, *N. Langsdorffii*) or even as single chromosomes (*N. rustica*).

In *Nicotiana rustica* I counted from one to 12 groups of chromosomes of various size during the interkinesis. Regular second metaphases were not formed. Dividing chromosomes get spread in the cytoplasm in groups of

various numbers (of one, two or several chromosomes). At the end of the phase that corresponds to the second anaphase, the chromosomes are spread abnormally in cytoplasms in groups of several chromosomes or even as single chromosomes. Each group or even each chromosome get surrounded with nuclear membrane, thus numerous nuclei (instead of 4) are formed. The cytoplasm contracts arround each nucleus or groups of closely situated nuclei, so

Fig. 1.

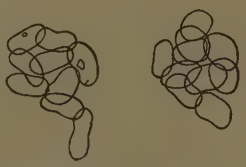

Fig. 3. Fig. 4. Fig. 5.

Fig. 2.

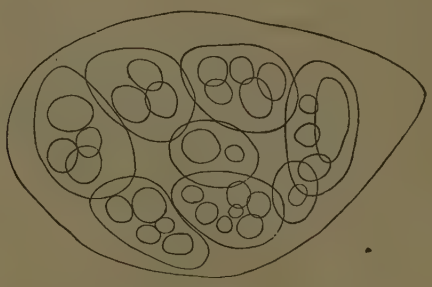

Fig. 6.

Fig. 1—6. Meiosis in tobacco pollen mother cells from floral buds treated with acenaphthene. 1, 4 and 5—in *Nicotiana longiflora* (n=10); 1—beginning of second division; 4—beginning of the first division which corresponds to first metaphase—10 bivalents; 5—end of the second division; 3—beginning of the first division which corresponds to first metaphase in *Nicotiana Sanderae* (n=9)—9 bivalents without polar orientation; 2—meiosis in *Nicotiana rustica*, only a part of the chromosomes are drawn, namely those which lied approximately at one level; 6—a pollen mother cell from *N. rustica* during the «tetrad» stage with 8 microspores, most of which have several nuclei of various size.

that a variable number of microspores is formed. Each microspore has one nucleus or more than one.

In *N. rustica* (n=24) I have found from 6 to 18 microspores per «pollen mother cell» during the «tetrad» stage. Some microspores had up to 6 nuclei. In all microspores of a pollen mother cell I counted a total of 31 nuclei (Fig. 6). In the species with smaller chromosome numbers much smaller numbers of microspores and nuclei in the microspores were found. *N. Langsdorffii* (n=9), *N. Sanderae* (n=9) and *N. longiflora* (n=10) formed even monads, unequal diads, triads, unequal tetrads, pentads, etc. The larger number of microspores in *N. Langsdorffii* was 9, in *N. longiflora* and *N. Sanderae*, 10; in *N. glauca* (n=12) I counted even 11, and in *N. Cavanillesii* (n=12)—12.

Comparing the effect of various agents upon the meiosis which I have had the opportunity to study during the last ten years, namely, various temperatures ([3,4]), fumigation with nicotine sulphate ([5]), woundings ([6]), various

parasites ([7],[8],[9]) etc., with the effect of acenaphthene sublimating particles, the latter are the most powerful agents to deviate the normal procedure of the meiotic (and mitotic) processes. In all six species treated with acenaphthene and studied cytologically I could not find a single pollen mother cell with normal meiosis.

X-rays also disturb meiosis, but they induce numerous chromosome dislocations and fragmentation. Acenaphthene does not seem to induce such kind of changes, or if they occur, they should be rare, since I could not find any indications for it.

Acenaphthene may serve as an excellent means for inducing chromosome aberrations (aneuploid), in which one or several whole chromosomes can be involved when treating floral buds or flowers (meiosis in the embryo-sac), and polyploidy when treating somatic tissues, especially, germinating seeds or embryos shortly after fertilization. Acenaphthene and colchicine experiments will also throw some light upon the dynamics of the cell division in general and meiosis in particular.

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GENETICS

INCREASED VIABILITY OF HETEROZYGOTES FOR A LETHAL IN *DROSOPHILA MELANOGASTER*

By R. A. MASING

(Communicated by N. I. Vavilov, Member of the Academy, 14.V.1938)

In our genetic and cytological studies of lethal mutations arising under natural conditions in the second chromosome of *Drosophila melanogaster* (6) we came across a very interesting and paradoxical fact consisting in the increased viability of flies heterozygous for the lethal as compared with those bearing no lethal.

All the lethals studied were balanced with the aid of the line *Cy*. As shown by the analysis (6), they were not connected with chromosomal aberrations, which falls in line with the data of Sakharov (8) and Demerec (2) who have proved that lethals spontaneously arising under laboratory conditions represent point mutations.

For the purpose of cytological analysis of the lethals in salivary glands, according to Painter, males $\frac{Cy}{l}$ (l — lethal) were crossed with females *ebony**; in order to check virginity of the females, flies produced from the cross were examined. It was noticed that in certain lines, instead of the expected ratio 1 *Cy* : 1 normal, a considerably larger number of normal flies than *Cy* was obtained. Repeated analysis has shown that this phenomenon reoccurs regularly, and 19 lethals from various geographic localities were studied in the same manner with regard to different viability of the flies $\frac{Cy}{+}$ and $\frac{l}{+}$. In nine lines a higher viability of heterozygotes for the lethal was discovered. No relation to definite geographic areas could be discovered: increased viability was observed in flies from different areas.

Aside from crossing males $\frac{Cy}{l}$ with females *ebony* in certain lines males $\frac{Cy}{l}$ were crossed also with females taken from a line which had been inbred during a long time, in order to find out whether the viability could be increased by a specific interaction between the gene *ebony* and the chromosome bearing the lethal.

As a control served the crossing of males $\frac{C}{+}$ both with *ebony* and normal females from the same inhered line. Attention is drawn to the fact (Table 1) that flies *Cy* showed in our experiment no increased viability in contrast to the

* The gene *ebony* is located in the third chromosome.

current opinion. In the experiments designed for localization of the lethals with the help of lines $\frac{Cy}{BlL}$ in F_2 a significant preponderance of flies Cy was observed compared with non-Cy flies. There seems to exist a special interaction between Cy, the genes Bl, L and the lethal studied (it is of interest that in the experiments of localization of lethals with the aid of the line $\frac{Cy}{SSp}$ no such predominance of Cy flies in F_2 was observed).

The results of crosses which yielded the most peculiar results are given in Tables 1 and 2.

Table 1

Cross	Tashkent 95				Control			
	$\frac{+\ e}{+\ e} \times \frac{Cy\ +}{l\ +}$		$\frac{+\ +}{+\ +} \times \frac{Cy\ +}{l\ +}$		$\frac{+\ e}{+\ e} \times \frac{Cy\ +}{+\ +}$		$\frac{+\ +}{+\ +} \times \frac{Cy\ +}{+\ +}$	
The genotype of the progeny obtained	$\frac{Cy\ e}{+\ +}$	$\frac{l\ e}{+\ +}$	$\frac{Cy\ +}{+\ +}$	$\frac{l\ +}{+\ +}$	$\frac{Cy\ e}{+\ +}$	$\frac{+\ e}{+\ +}$	$\frac{Cy\ +}{+\ +}$	$\frac{+\ +}{+\ +}$
Observed	479	649	684	983	496	530	518	521
Expected	564 ±16.8	564 ±16.8	833.5 ±20.4	833.5 ±20.4	513 ±16.0	513 ±16.0	519.5 ±16.0	519.5 ±16.1
$\frac{\text{Diff.}}{m}$	5.1		7.3		1.06		0.09	

Table 2

Cross	Kuibyshev 73				Tashkent 201			
	$\frac{+\ e}{+\ e} \times \frac{Cy\ +}{l\ +}$		$\frac{+\ +}{+\ +} \times \frac{Cy\ +}{l\ +}$		$\frac{+\ e}{+\ e} \times \frac{Cy\ +}{l\ +}$		$\frac{+\ +}{+\ +} \times \frac{Cy\ +}{l\ +}$	
The genotype of the progeny obtained	$\frac{Cy\ e}{+\ +}$	$\frac{l\ e}{+\ +}$	$\frac{Cy\ +}{+\ +}$	$\frac{l\ +}{+\ +}$	$\frac{Cy\ e}{+\ +}$	$\frac{l\ e}{+\ +}$	$\frac{Cy\ +}{+\ +}$	$\frac{l\ +}{+\ +}$
Observed	473	656	403	418	385	379	423	745
Expected	564.5 ±16.8	564.5 ±16.8	410.5 ±14.3	410.5 ±14.3	382 ±13.8	382 ±13.8	574 ±17.0	584 ±17.0
$\frac{\text{Diff.}}{m}$	5.4		0.5		0.29		9.47	

The individual peculiarities of the lines referred to are apparent in the tables.

The lethal Tashkent 95 (Table 1) gives in heterozygous condition an increased viability both in crosses with *ebony* and with normal females. The difference between observation and expectation is absolutely significant in the two cases (in the cross with *ebony* females, $\frac{\text{Diff.}}{m} = 5.1$, and with normals, $\frac{\text{Diff.}}{m} = 7.3$).

The lethal Kuibyshev 73 (Table 2) gives increased viability only in the cross with *ebony*. Cross with normal females gives two classes of flies with equal viability.

The lethal Tashkent 201 (Table 2) is characterized by the opposite phenomenon, viz., increased viability is observed in heterozygous flies only in crosses with normal flies, it being very great in this case: $\frac{\text{Diff.}}{m} = 9.47$.

In the first instance (Kuibyshev 73) the presence of *ebony* in heterozygous condition increases the viability of the flies bearing the lethals, while in the second it hinders the expression of increased viability which, as it was noted, is very sharply expressed in crosses with normal flies.

As may be easily seen, the relations prove to be very peculiar.

We imagine three possible explanations of the phenomenon described.

1. The lethal gene under investigation may be a dominant viability gene with a recessive lethal effect. The majority of dominant genes in *Drosophila* are lethal in homozygous condition, e. g., *Bl*, *S*, etc.

2. The lethal gene when arising is deleterious for the life of the individual, and the increased viability observed by us under laboratory conditions is the result of evolutionary accumulation of modifying genes which neutralize the effect of lethals in nature.

3. The lethal gene is accidentally linked with a gene of increased viability.

The first hypothesis of the dominant gene lethal in homozygous condition is borne out by the fact that a similar phenomenon of increased viability of individuals heterozygous for the lethal was discovered by us among lethals which had arisen under laboratory conditions both spontaneously and under the influence of chemicals. It is clear that modifying factors which might accumulate in the process of evolution are out of question in this case.

The second hypothesis is supported by the fact that in some of the lines studied a gradual decrease of viability is observed.

Table 3 contains data for the lethal Andijan 95 which illustrate this situation.

Table 3

Cross	Andijan 95			
	May 1937		November 1937	
	$\frac{Cy +}{l +} \times \frac{+ e}{+ e}$		$\frac{Cy +}{l +} \times \frac{+ e}{+ e}$	
The genotype of the progeny obtained	$\frac{Cy\ e}{+\ +}$	$\frac{+\ e}{l\ +}$	$\frac{Cy\ e}{+\ +}$	$\frac{+\ e}{l\ +}$
Observed	207	337	338	489
Expected	272 ±11.7	272 ±11.7	438.5 ±14.8	438.5 ±14.8
$\frac{\text{Diff.}}{m}$	5.6		3.4	

In May 1937 crosses of this line with *ebony* gave $\frac{\text{Diff.}}{m} = 5.6$, in November of the same year $\frac{\text{Diff.}}{m} = 3.4$, i. e. considerably less. Inasmuch as all the lines were balanced with the aid of *Cy*, and *Cy* does not completely

inhibit crossing-over in the chromosome II, it was possible to suppose that modifiers which increase viability are gradually dispersed owing to low struggle for life under laboratory conditions.

The third hypothesis, viz., that the lethal gene is accidentally linked with a gene for increased viability, seems to be hardly probable. Non-lethal genes of increased viability are probably of very rare occurrence. One case of this kind has, it is true, been described referring to such a mutation in the sex-chromosome, after treatment with X-rays. In studying for lethals 150 chromosomes of the Crimean population and 170 chromosomes treated with X-rays we specially looked for genes of increased viability, but were unable to find a single case.

The data reported by us on increased viability of flies heterozygous for lethals appear to us to be of great interest, especially taking into account that they were discovered in nature.

As we have as yet studied the chromosome only as a whole, we cannot insist upon the connection of the increased viability discovered by us with a lethal gene; we nevertheless find it possible to assume that a lethal in heterozygous condition is not always indifferent for the life of the individual, and that occasionally—which sounds rather like a paradox—it may even increase the viability.

Of no less interest appears to us the fact, that the gene *ebony*, rather indifferent from the point of view of viability, may even in heterozygous condition exert a material effect upon the viability of the individual.

It is known ([9],[3],[4],[5]) that natural populations of *Drosophila* are highly saturated with lethal and non-lethal genes in heterozygous condition.

In the light of our data selection for heterozygotes appears to play an important part in the dynamics of populations; genetico-anatomic processes are thus pushed into the background. It is, of course, important to establish, how frequent such a phenomenon is in nature (probably, not so rare, since out of 19 lethals studied 9 proved to possess increased viability), and whether it is actually connected with heterozygoty for the lethal. If it proves to be true, the question raised by Altschuler, Borisenko and Poliakov ([1]) as to the expedience of taking for the final and desirable goal of breeding the production of a homozygous line, as well as the question on the use and harm of inbreeding would appear under a new aspect. Lack of space does not permit of a fuller discussion of this subject.

It is of interest to note that a similar lethal gene which increases the viability of flies heterozygous for it, as compared with normal ones bearing no lethal, was described in 1937 by Nabours and Kingsley ([7]) in the grasshopper *Apotettix eurycephalus*.

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TAXONOMY

SOLANUM BOERGERI BUK. — A NEW POTATO SPECIES FROM URUGUAY

By S. M. BUKASOV

(Communicated by N. I. Vavilov, Member of the Academy, 14.V.1938)

Area and Position in Taxonomic System. Uruguay is the home of the classical wild potato, *Solanum Commersonii* Dun. The area of this species has been given as the La Plata basin and the region at the mouths of the Uruguay and Parana rivers. The plant described by Dunal originated from the vicinity of Montevideo. Such live specimens of wild potatoes from Montevideo ranked as *S. Commersonii*, as are now known, have been found to be triploids.

During the course of the past few years the Institute of Plant Industry (Leningrad) has obtained, through the kindness of Senor Teofilo Henry, agronomist, of Estanzuela (Uruguay) a number of new specimens of wild potatoes from Uruguay, the study of which has given us a clearer understanding of the wild potatoes of that country. It now appears that the area of the triploid species from Montevideo, for which we propose to keep the original name *S. Commersonii* Dun. s. str., is very limited, apparently not extending beyond the direct vicinity of Montevideo. To the east, at the mouth of the La Plata on Goritti Island opposite the town of Maldonado, there is found another species: *S. Ohrondii* Carr., a diploid form easily distinguishable morphologically both from *S. Commersonii* s. str. and from western species of this group: *S. Henryi* Buk. and Lech. from Estanzuela, *S. sorianicum* Buk. from the Departments of Soriano, and others whose descriptions have not yet been published. We are unacquainted with diploid species from Montevideo, though apparently they exist, judging by the work of Gassner in which he mentions species from that section producing seed-balls, which triploid forms fail to do. Diploid species on the Argentine shore of the La Plata are also unknown to us in the live state, with the exception of one species *S. laplaticum* Buk., differing somewhat from the previously mentioned species and possibly growing in the vicinity of Buenos Aires.

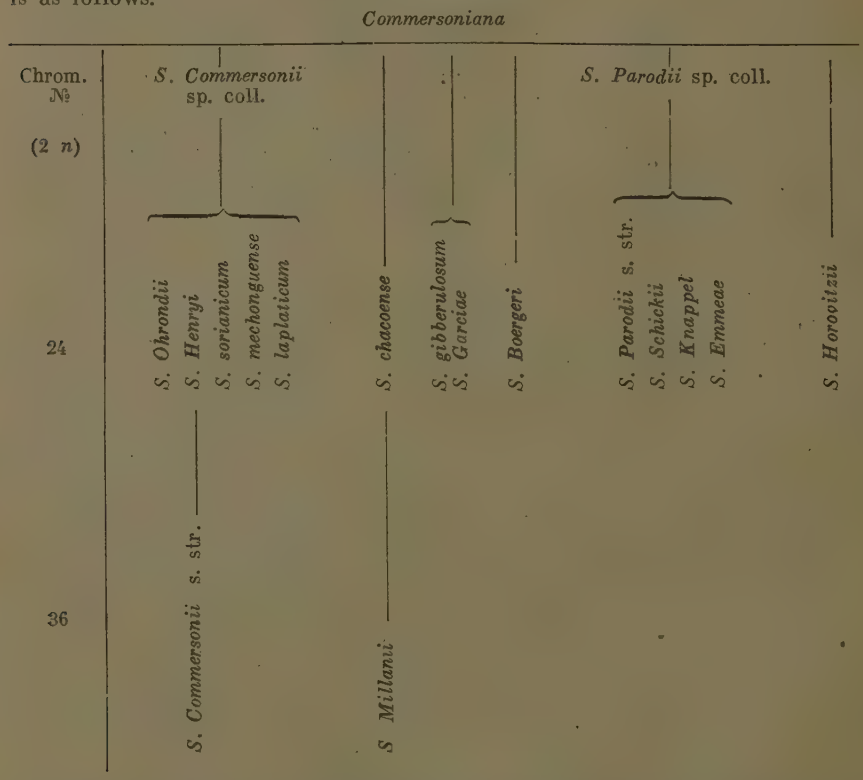
Hence, all references in the literature to the finding of *S. Commersonii* outside the direct vicinity of Montevideo, e. g. at Mercedes or Buenos Aires, concern presumably not triploid *S. Commersonii* s. str., but an entire group of quite closely related species which have been assembled by us in *S. Commersonii* sp. coll.

Species taxonomically distinct from *S. Commersonii* sp. coll. were not known to exist in Uruguay. The recent discovery among the wild potatoes of the Department of Soriano of a species quite distinct from *S. Commersonii*

was, therefore, of great interest. This species we have named *S. Boergeri* in honour of Professor A. Boerger. The discovery was all the more interesting, since the new species originated from the same locality as *S. sorianicum*, a species closely related to *S. Henryi*, i. e. belonging to *S. Commersonii* sp. coll.

As regards its morphological characters, *S. Boergeri* stands closer to the mountain group originating in the Argentine Andes and headed by *S. Parodii* sp. coll. (sometimes referred to in the literature under the incorrect name, *S. Caldasii* var. *glabrescens*) from which it differs by its greater pubescence, broad leaf lobes, and other characters. *S. Horovitzii* Buk., taxonomically close to *S. Parodii* and also densely pubescent, differs from *S. Boergeri* by reason of its unique, broad stem ribs, not duplicated in any other wild or cultivated form of potato known to us.

The position of *S. Boergeri* in the taxonomic system of South American potatoes with stellate corolla as known to us in the live state and placed by us in a special group, *Commersoniana*, apart from the Mexican *Pinnatisecta*, is as follows:



Description of *Solanum* (*Tuberarium*) *Boergeri* Buk. Tubers on long stolons*, small, no larger than haselnuts, yellow lenticels not distinct**.

* Some of the stolons come out on the surface of the soil and give rise to daughter plants.

** This species is characterized by an exceptionally long dormant period (even in the greenhouse the tubers do not sprout until July).

Plants, when grown in the open near Leningrad, short, about 30 cm in height, with short internodes, about 1 cm long; when grown in the greenhouse, tall, of rapid growth, with long internodes, shape not at all rosette-like.

Stems with narrow ribs, which are less than 1 mm wide and either straight or only slightly crimped.

Leaves interruptedly odd-pinnate, with 5—6 pairs of primary lobes, gradually diminishing in size toward the base of the leaf; between the primary lobes small, secondary lobes, one pair between the terminal lobe and the first pair of primary lobes and from one to two pairs between the first and second pair and between the second and third pair of primary lobes; further toward the base of the leaf there is usually not more than one pair of secondary lobes, but there may also be one pair of tertiary lobes; one pair of the latter may also occur between the second and third pair of primary lobes. The total number of segments per leaf in the central part of the plant ranges from 15 to 31.

Length of leaf, 17—27 cm; lobes usually broad* as in *S. tuberosum*, the size of the terminal ones being 60—62×41—50 mm; petiole short, 25—35 mm long; ansae (secondary petioles), 10—15 mm long; auriculae large, 27×17 mm.

Very short pubescence on both upper and lower surface of leaves along the veins and between them.

Flower clusters located at different heights; the lowest peduncle branches from the stem at a height of 8—9 cm (at the 9th—11th node); length of peduncle, 6.5—15 cm. Branches in a cluster number as many as 4 or 5. Distribution of flowers on a branch usually normal, as in most *Tuberarium* species; occasionally, however, after one or two flowers branching from the rachis at intervals, the other pedicels branch from a single point, as is frequently the case in *S. Commersonii* Dun. and more rarely in *S. Schickii* Juz. and Buk. and *S. gibberulosum* Juz. and Buk.

The buds sometimes drop without opening.

Length of the pedicels of the first flowers of a branch: below the joint, 3—13 mm; above it, 11—15 mm, i. e. the joint is situated below the middle of the pedicel, sometimes fairly close to its base.

Calyx symmetrical, short, 4—5 mm, with very short mucrons (about 0.5 mm). Pubescence of calyx very short. Corolla stellate, white; longer radius, 13—15 mm; shorter radius, 7—10 mm; width of lobes, 5—11.5 mm; length of lobes, 7—9 mm; length of acumen, 5—7 mm.

Anthers small, 4.5—6×1.5 mm, forming a cylindrical or somewhat oval column; filaments slender. Style quite long, 10—12 mm, extending above the stamens 2.5 to 6 mm, slender, with a small rounded (rarely 2-lobed) stigma. Seed-balls are formed. The diploid chromosome number is 24 (according to E. V. Ivanovskaya). Susceptible to *Phytophthora infestans*.

Native of Uruguay, Department of Soriano, Agraciada, 1935, collected by Senor Teofilo Henry, agronomist; grown from seed sent by him in experimental field near Leningrad.

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* The lobes are narrower in the upper part of the plant; they also vary in width in different seedlings.

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PLANT PHYSIOLOGY

DEGENERATION OF POTATO CULTIVATED IN THE SOUTH

By J. V. RAKITIN and P. M. SHUMOVA

(Communicated by A. A. Richter, Member of the Academy, 10.V.1938)

The question as to the degeneration of potato in the south is not a new one. But in spite of its being much discussed and studied by many investigators, it is only in these recent years that the problem was solved, owing to the works of Lyssenko (1). He has shown that the principal reason potato degenerates in the south lies in the high temperature prevailing during the period of tuber-formation. It has, thus, become clear what we have to do to prevent this evil. Such agrotechnical measures should be endeavoured as would permit southern crops to escape the harmful influence of high temperatures. The practical solution of the problem was achieved by Lyssenko by means of summer planting. When potato is set in summer, the formation of tubers falls within the period of moderate temperatures prevailing at the end of summer—beginning of autumn.

In the present article we purpose to throw some light on the data obtained while subjecting potato to the effect of high temperature under laboratory conditions. The experiment was taking place during the vegetation period of 1936.

Respiration of Tubers. Respiration was estimated in young tubers, dug out on August 21. Previous to estimation, part of them was kept at 4° C for 24 hours; others, for the same period of time, were treated with a temperature of 20°C. This preliminary treatment being accomplished, the tubers were placed in glass receivers, and the air deprived of carbon dioxide and heated up to 40 and 20° C respectively was forced through. Catching of carbon dioxide ordinarily lasted 2 hours, being effected by means of a Richter absorber (2). The results of one of the typical experiments are referred to in Table 1.

Table 1

Variety	Temperature, °C	Carbon dioxide of respiration in %
Epicurus	20	100
	40	215
Early Rose	20	100
	40	225

About the same intensifying of respiration under the influence of high temperature has been observed by some other authors too.

Formation of Ethyl Alcohol and Acetaldehyde. Alcohol and aldehyde were determined in the tubers kept for 24 hours at respective temperatures. The estimation of alcohol was done according to Nicloux (5); that of aldehyde, according to Agabalyantz and Sovenkova, the latter method being modified by the author of the present article (3). In Table 2 the content of alcohol and aldehyde in the tubers of the 20° variant is taken for 100.

Table 2

Sprouting Tubers Harvested in 1935

Variety	Temperature, °C	Ethyl alcohol in %	Acetaldehyde in %
Epicurus	25	100	100
	40	115	180
Early Rose	25	100	100
	40	118	200

Content of Ammonia and Amides. Ammonia and amides were estimated in tubers kept for 24 hours at 25 and 40° C respectively. The ammonia was estimated according to Longi (6); the amides, according to Sachsse-Vickery (7).

The content of ammonia and amides in tubers kept for 48 hours at low temperatures is taken for 100.

Table 3

Variety	Temperature, °C	NH_3 in %	Amides in %
Sprouting tubers harvested in 1935			
Epicurus	23	100	100
	40	275	55
Early Rose	23	100	100
	40	200	60
Young tubers harvested in August 21, 1936			
Epicurus	23	100	100
	40	380	58
Early Rose	23	100	100
	40	390	54

An analogous change in the ratio of amide and ammonia nitrogen due to the effect of high temperature has been observed in potato sprouts by Altergott (4).

The content of NH_3 in the leaves of Early Rose is shown in Table 4. The leaves with their petioles immersed in water were kept at 25 and 40°C respectively during 2 hours.

Table 4

Temperature, °C	Content of NH_3 in %
25	100
40	158

Table 5 shows the content of NH_3 in the leaves of Early Rose grown in a special thermostat with a temperature of 38—40°C and in the experimental plot at a temperature of 20—27°C.

Table 5

Temperature, °C	Content of NH_3 in %
20—27	100
38—40	202

Effect of High Temperature upon Tuber-Formation. The plants of Early Rose under test were growing in iron vegetative vessels. The tubers were set in the vessels on June 4. All plants in vessel were left to stand on a greenhouse plot, until the formation of flower buds set in. Then they were divided into four groups. One group remained on the

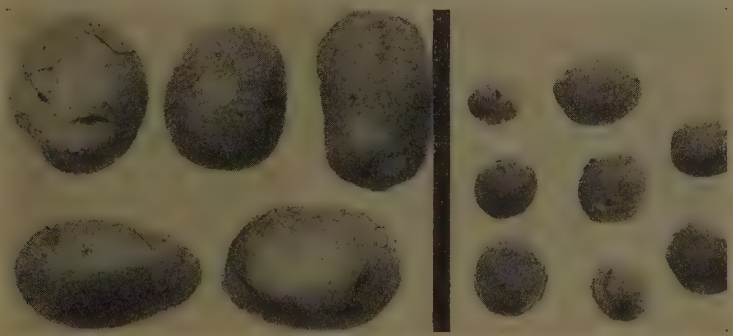

Fig. 1. Left—potato tubers grown at room (ordinary) temperature; right—potato tubers kept in thermostat (40—41° C) for 10 days.

plot while the three other were put into a thermostat in a hothouse. Only the underground parts of the plants were placed in the thermostat. The temperature in the latter was automatically maintained within 40—41°C. The temperature of air in the hothouse varied within 20—30°C, while that of the plot was somewhat lower throughout the experiment. The plants of all variants were given tap water every day. A 75% moisture was imparted to soil by each watering. The plants of the other group were kept in the thermostat for 10, 25 and 50 days (i. e. until September 1) respectively. Then the vessels were transferred to the greenhouse plot and remained there until the end of the experiment. On September 1 it was established that the longer the plants had been exposed to the influence of high temperature, the more expressed were the changes in tubers and overground parts. The changes in overground parts consisted in a rather strong depression of growth and in reduced dimensions of morphological elements. As to the tubers, if they had been subjected to a short influence of high temperature, they showed a marked

decrease in size, while after a long treatment pronounced signs of degeneration could be observed (abnormal, resp. spindle-like shape of tubers, swelling at the base of the eyes, mosaic of the periderm) (Fig. 1, 2).

Besides this, the tubers grown under conditions of enhanced moisture had their eyes prematurately sprouted. Eyes sprouted the more rapidly the longer was the period during which the forming tubers suffered the action of high temperature. From the data obtained it may be seen that high temperature

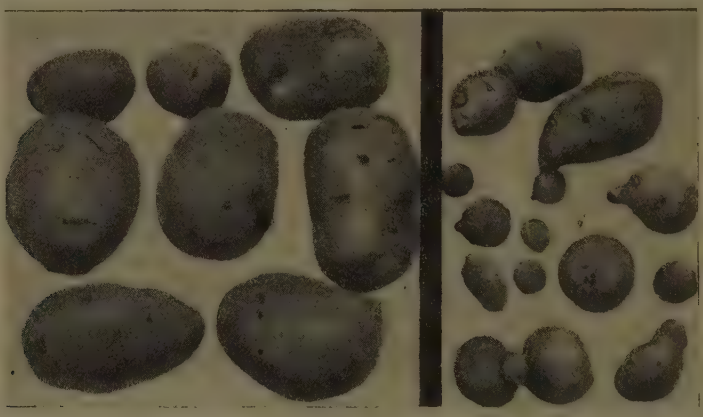

Fig. 2. Left—potato tubers grown at reduced (normal) temperature; right—potato tubers grown in thermostat at 40—41C°.

intensifies respiration in potato, promotes the formation of products of anaerobic metabolism (C_2H_5OH, CH_3CHO), reduces the content of amides and enhances the formation of ammonia. In other words, we find the disintegration processes to be materially promoted by high temperature, this being the most typical feature of the degeneration of potato crops under southern conditions.

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PHYSIOLOGIE VÉGÉTALE

LE «SÉCHAGE PHYSIOLOGIQUE» DU BOIS DU HÊTRE

Par A. A. YATSENKO-KHMELEVSKY

(Présenté par A. A. Richter, de l'Académie, le 24.IV.1938)

L'humidité élevée du bois fraîchement coupé doit être considérée comme un inconvénient grave pour le forestier, car elle alourdit le bois, fait dans quelques cas le flottage impossible et agit d'une manière négative sur les propriétés techniques du bois (gercement durant le séchage). Par conséquent, il est naturel que les forestiers cherchent depuis longtemps les moyens d'obtenir dans le taillis même le bois plus ou moins sec.

Ces dernières années il s'est propagée dans la pratique forestière de l'Union Soviétique la préparation du bois pour le flottage par la méthode de l'annélation (ceinturage). Cette opération est appliquée surtout au bois du hêtre ([1], [2]) et du melèze ([3]), car le bois lourd de ces essences est très difficile à flotter. Pour obtenir le bois plus ou moins sec, on procède de la manière suivante: l'arbre désigné à être coupé est scié près de la base à une profondeur égale à un tiers du rayon de la section du tronc. On présume que par ce fait l'ascension de l'eau doit diminuer et, la transpiration de la cime restant la même, on pourra obtenir dans le tronc un déficit de l'eau, grâce à quoi, après l'abattage, le bois sera plus sec. Si après la coupe de l'arbre le feuillage n'est pas coupé, la transpiration des feuilles doit encore augmenter ce déficit de l'eau.

Une méthode semblable est pratiquée assez largement aux États-Unis dans l'exploitation du cyprès chauve (*Taxodium distichum*) ([4]).

Ces opérations sont désignées souvent par des termes très différents (annélation, ceinturage; anglais—girdling, deadening; allemand—Ringelung; russe—подвяливание, кольцевание). Il semble plus commode pourtant d'appliquer ici le terme «séchage physiologique du bois» plus apte à exprimer la nature de ce processus.

L'importance de cette méthode pour les exploitations forestières a amené l'Institut des Recherches Forestières de la Transcaucasie à organiser au courant de l'année 1935 une série d'expériences sur le séchage physiologique du bois du hêtre. Les travaux ont été poursuivis dans le massif d'Akhméty dans l'économie forestière de Kakhétie (République Soviétique Socialiste de Géorgie). La communication présente est un exposé bref des résultats de ces travaux.

Les expériences ont été organisées deux fois, vers la fin de mai et au milieu de juin 1935. Les méthodes employées ont été empruntées au Prof. Junitzky ([1]), c'est-à-dire les arbres expérimentaux ont été sciés à la base, à 0.3 m du niveau du sol, ou par toute la périphérie du tronc, à une profondeur de

3—5 cm, ou de deux côtés opposés, un trait de scie un peu plus bas que l'autre, chaque profondeur de coupe égalant un tiers du rayon. En outre, une série d'arbres a été coupée et laissée dans le taillis avec leurs cimes intactes.

Les arbres sciés, après un laps de temps déterminé, ont été abattus. Une série de ces arbres a été débitée immédiatement après la coupe pour pouvoir prendre des échantillons, une autre série a été laissée gisante dans le taillis avec la cime intacte.

L'influence relative des méthodes diverses du séchage physiologique était estimée de la manière suivante: après un laps de temps déterminé, le tronc de l'arbre séché était débité pour pouvoir prendre des échantillons à examiner. Les échantillons étaient pris en forme de rond, haut de 3—5 cm, pris dans toute la section du tronc. Ces ronds étaient pris à une distance de 0.5, 1.0, 1.5, 5.0 et 7 m du gros bout du tronc. De chaque rond on prenait à la hache deux échantillons pour la détermination de l'humidité du bois, un à la périphérie du rond (dans l'aubier), l'autre au millieu du rayon (dans le bois parfait). Les échantillons étaient immédiatement placés dans des vaisseaux d'aluminium fermés hermétiquement.

Simultanément avec l'analyse des grumes et des troncs expérimentaux on déterminait l'humidité du bois des arbres sur pied qu'on abattait à ce but, et des grumes, laissées dans le taillis dans les conditions ordinaires pour les mêmes laps de temps que les troncs expérimentaux.

En tout nous avons examiné trente-neuf arbres et dix arbres ont été coupés pour servir de contrôle. La durée et les méthodes du séchage employées sont indiquées dans la table 1. Les changements survenus dans l'humidité du bois après le séchage sont rassemblés dans les tables 2—4*.

Table 1

Méthode et durée du séchage des arbres expérimentaux

	Terme	Date du sciage	Date de l'abattage	Temps du séchage sur pied (jours)	Date de la prise des échantillons	Temps du séchage par la cime
Les arbres sur pied séchés par des traits de scie à la base du tronc	I	30.V	15.VII**	45	—	—
	II	20.VII	21.X**	90	—	—
Le séchage combiné . . .	I	30.V	15.VII	45	5.X	80
	II	20.VII	24.X	64	24.X	30
Le séchage des arbres abattus par la cime*** . .	I	—	18.VII	—	18.X	90
	II	—	30.V	—	28.VII	60

La table 2 démontre clairement que les traits de scie effectués à la base des arbres sur pied, dans un délai de trois mois, n'exercent presqu'aucune influence sur l'humidité du bois. Nous présumons que cela s'explique par le fait que le hêtre est une essence privée de duramen, dont les couches intérieures du tronc (le bois parfait—Reifholz des auteurs allemands) sont anatomiquement capables à transmettre l'eau—ne sont pas fermées par des thylles.

Le bois des arbres coupés et laissés gisant avec leurs cimes montre un état bien différent (tables 3—4). L'abaissement de l'humidité dans ce cas-là

* Nous donnons ici les tables les plus caractéristiques.

** Les échantillons ont été pris le même jour.

*** Quelques expériences supplémentaires du séchage avec la cime ont été exécutées avec des arbres de contrôle.

Table 2

L'humidité comparée du bois des arbres sciés à la base et laissés sur pied trois mois (du 20 juillet jusqu'au 21 octobre 1935)*

	La hauteur de la souche, m	L'aubier	Le bois parfait
Les arbres de contrôle abattus le 25 juillet .	0.5	62.5	74.3
	4.5	66.0	63.3
Les arbres sciés de deux côtés opposés . . .	0.5	70.6	67.6
	1.0	69.1	75.6
	1.5	89.6	—**
Les arbres sciés par toute la périphérie du tronc	0.5	59.9	—
	1.0	73.9	—
	1.5	71.9	—
Les arbes de contrôle abattus le 23 octobre .	0.5	66.6	64.1
	4.5	71.2	77.6

Table 3

L'humidité comparée du bois des grumes débitées et des arbres laissés dans le taillis avec la cime pour des termes différents

Le jour de l'abattage	Le jour de la prise des échantillons	La durée de l'expérience (jours)	Le caractère de l'objet de l'expérience	La distance du bout de la grume ou du tronc	L'aubier		Le bois parfait
5.VII	25.VII	20	La grume	Le bord	80.7		64.3
				0.5		77.7	
			Le tronc avec la cime	Le bord	44.5		47.7
				0.5	33.0		42.5
30.V	28.VII	58	La grume	Le bord	46.7		50.4
				0.5		77.6	
			Le tronc avec la cime	Le bord	42.3		43.1
				0.5		52.2	
13.VII	4.XI	111	La grume	Le bord	44.9		—
				0.5	94.6		65.0
				2.0	101.2		80.8
			Le tronc avec la cime	0.5	54.1		51.3
				2.0	86.0		57.9
30.V	8.X	128	La grume	Le bord	61.8		—
				0.5	88.4		—
				2.0	94.2		63.0
			Le tronc avec la cime	0.5	39.6		—

* Les données représentent les moyennes de deux déterminations.
** Les déterminations n'ont pas été faites.

Table 4

L'humidité relative du bois des arbres laissés sur pied—sciés à la base—durant un mois et demi (du 30 mai jusqu'au 15 juillet 1935) et après laissés gisant avec la cime intacte durant 80 jours (du 15 juillet jusqu'au 5 octobre 1935)*

Le caractère de l'expérience	L'état de la cime	La distance de la souche	L'aubier	Le bois parfait
Le tronc est privé de l'écorce à la distance de 8 m	La cime est intacte (au moins 2/3 sont conservés)	Le bord	34.4	39.9
		1.0	35.5	43.1
		4.0	40.0	50.9
		7.0	50.5	53.8
	La cime est brisée plus de 2/3	0.5	39.5	39.1
		1.0	43.0	43.0
		4.0	42.9	49.6
		7.0	50.9	57.0
Le tronc est couvert de l'écorce	La cime est intacte (au moins 2/3 sont conservés)	0.5	53.0	51.5
		1.5	46.7	40.6
		4.0	54.8	53.0
		7.0	57.1	51.7
	La cime est brisée plus de 2/3	0.5	65.7	58.6
		1.0	75.4	52.3
		4.0	68.6	58.1
		7.0	70.0	58.2

est très marqué et doit être entièrement attribué à l'influence desséchante de la cime, comme le démontre l'humidité du bois plus élevée chez les arbres avec leurs cimes brisées pendant la chute.

L'effet général du séchage physiologique est assez important car, comme nous l'avons vu, l'humidité du bois diminue de 70% jusqu'à 40—45%, ce qui baisse le poids spécifique du bois de 0.98 jusqu'à 0.75—0.76, ce qui correspond à une diminution du poids d'un mètre cube de 220 kg, c'est-à-dire presque de 25%.

En résumant tout de qui a été exposé plus haut, il faut noter les conclusions suivantes: 1) le sciage des arbres sur pied n'a presque aucune influence sur l'humidité du bois; 2) la conservation de la cime sur l'arbre abattu, au contraire, provoque un desséchement considérable du bois (l'humidité du bois tombe de 65—75% jusqu'à 40—45%), ce qui abaisse le poids spécifique du bois de 0.98 jusquà 0.75—0.76).

Néanmoins, l'application de cette méthode à l'exploitation du hêtre dépend surtout de l'état du bois pendant la durée du séchage. Nos observations à ce sujet seront exposées dans une communication suivante.

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* Toutes les données représentent les moyennes de 4—5 déterminations; l'humidité du bois des grumes de contrôle voir dans la table 3.

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PLANT PHYSIOLOGY

VERNALIZATION OF GRAIN AT THE TIME OF THE FORMATION OF THE EMBRYO

By N. P. GLINYANY

(Communicated by N. I. Vavilov, Member of the Academy, 8.V.1938)

In 1935 there appeared an interesting paper by Kostyuchenko and Zarubailo on «Natural Vernalization of Grain during Ripening» (1). These investigators found that plants of the same varieties of winter wheat, but derived from seeds of different origin, greatly differed in their development. Plants grown from seeds of the southern (Kirovobad) reproduction were on September 9 at the tillering stage, while plants of the same varieties derived from seeds of the northern (Khibiny) reproduction had by that time headed. Basing themselves on Lyssenko's theory, they consider that «the cause of this phenomenon... lies in the difference of the conditions under which the seeds used in these sowings were forming and ripening». In the north the ripening grain, being affected by low temperature, is, in their opinion, vernalized during ripening. They arrived therefore at the conclusion that in the plants derived from seeds of northern reproduction the cycle of development had been partly realized while the seeds were on the mother plants, owing to which they developed more rapidly than those grown from seeds of southern reproduction. The following year (1936) Gregory and Purvis (3) reported that they had succeeded to vernalize ripening grain of winter rye under artificial conditions. In 1937 (2) Kostyuchenko and Zarubailo confirmed their previous conclusion. This year Gregory and Purvis (4) have published new data showing that the embryo is more responsive to low temperature at early stages of development than at late stages.

Our own investigation was devoted to the problem of how seeds at different stages of ripeness respond to low temperature. In 1936 we first studied the length of the vernalization stage in a Spanish form of winter wheat *(Triticum vulgare* Vill. var. *albidum* Al.)*. We found that this wheat was not uniform as regards its vegetative period. Heading proved to be the more disorderly, the shorter was the period of vernalization. Not all of the plants grown from seeds vernalized for 18—20 days did produce heads, while the control plants failed to head altogether.

Under-vernalized plants, the development of which was greatly retarded, were taken for our experiments. Plants with many stems at different stages of ripeness were chosen. From each plant the ripest head was cut off as a con-

* The seed was obtained from the Wheat Section of the Institute of Plant Industry.

trol*. After cold weather had set in (September 21, 1936), the stage of ripeness of the other heads was registered. These heads were harvested on October 10, since on that day there occurred the first appreciable frost (—4°C). According to the data of the Pushkin Meteorological Station of the Institute of Plant Industry, the temperature of the air was during the period September 21—October 10 very favourable for our experiments **. The maximum temperature for these 20 days averaged 9.5° C.; the minimum temperature 2.2° C. Low temperatures were observed prior to September 21 too.

Unfortunately, it was not possible to put the plants under conditions favourable for the ripening of grain. Consequently, some of the heads produced grain incapable of germination. No seeds were obtained from heads whose anthers by September 21 had not matured, or but scarcely matured. On October 11 the total of heads on each plant were inspected, and it was found that during the twenty days since the first inspection (September 21) ripening had scarcely advanced at all. Grains of the same head were found to differ in ripeness.

The period during which the ripening plants were grown at a low temperature proved to be too short to enable us to reveal differences in response, just as it is impossible to reveal such differences by vernalizing germinated seeds for periods too short for the given variety. To obviate this difficulty, prior to sowing in spring 1937 the seeds from all the plants were germinated and subjected to vernalization for 10 days. Sowing was done on May 29 in the experimental field of the Peterhof Biological Institute. The grain from each head was planted in separate rows arranged according to the state of ripeness of the heads (as noted on September 21, 1936). The sprouts made their appearance from June 4 to June 7. For sake of convenience we have designated the families obtained by numbers 7342, 7343, 7344 qnd 7346 respectively ***. In all families the control plants were greatly retarded in development, as compared with the plants grown from the seeds which had been subjected to low temperature during ripening (from September 21 to October 10, 1936). This agrees with the conclusion of Kostyuchenko and Zarubailo ([1]) that seeds ripening at low temperature are being vernalized. Besides our experimental data show that seeds are the more responsive to the action of low temperature, the less ripe they are at the time cold weather sets in. Thus, in Family 7344 those plants developed most rapidly which were derived from seeds subjected to low temperature at the early milk-ripe stage, next in order were the plants from seeds at the milk-ripe stage, then the plants from seeds at the early yellow-ripe stage, while the control plants were developing most slowly (the table). As a result the plants in the plot where Family 7344 was sown gave a gradually ascending «curve» (Fig. 1).

The data for Families 7346 and 7343 agree in general with those for Family 7344. They also show that at early stages of ripeness seeds get vernalized more intensively that at late stages. Plants belonging to these families, obtained from seed at the early yellow-ripe and milk-ripe stages, developed more

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* From plant № 1 the control head was cut off on September 15 at the late yellow-ripe stage; from plant № 2—on September 15 at the full-ripe stage; from plant № 4—on September 12 at the late yellow ripe stage; from plant № 5—on September 14 at the late yellow-ripe stage.

** In 1936 our experiments were conducted at the Pushkin Experiment Station (near Leningrad) of the Institute of Plant Industry.

*** Owing to poor germination of the seed Family 7345 was dropped from the experiment.

rapidly than plants derived from seeds at the yellow-ripe stage*. Yet we failed to disclose here the same regularity as was observed in Family 7344. While in Family 7344 plants derived from seeds at the early yellow-ripe stage differed greatly from those grown from seeds at the milk-ripe stage, there was no marked difference between such plants in Families 7346 and 7343.

Fig. 1. Family 7344: *1*—plants from seeds of the control head, harvested September 12 at the late yellow-ripe stage; *2*—plants from seeds of a head which by September 21 had reached the yellow-ripe stage (harvested October 10); *3*—plants from seeds of a head which by September 21 had reached the milk-ripe stage (harvested October 10); *4*—plants from seeds of a head which by September 21 had only reached the early milk-ripe stage (harvested October 10). Photographed August 6, 1937.

In Family 7342 only the control plants displayed a retarded development, while the other plants, derived from seeds at the early yellow-ripe, late milk-ripe, and milk-ripe stages, did not differ materially.

On September, 1937, the control plants of Families 7346, 7342 and 7343 began to differ markedly from those of Family 7344. By September 26 the control plants of Family 7344 were at the tillering stage (only one of them was at the early shooting stage). By this time most of the control plants

* The «stage» indicated denotes, in each case, the stage of ripeness attained by a given head by September 21, 1936. The data for Families 7343 and 7346 point out that grain which on that date was classified as at the yellow-ripe or even late yellow-ripe stage was partly vernalized. But we do not conclude therefrom that also at these late stages of ripeness the grain is capable of being vernalized. Prior to September 21 low temperatures had also been observed, and, consequently, this grain may have been partly vernalized at a less ripe stage. This holds true for other cases too.

Data for Family 7344

	Head № 1 (control)	Head № 2	Head № 3	Head № 4
Date of harvest (1936)	Sept. 12	Sept. 12	Oct. 10	Oct. 10
Stage of ripeness on September 21, 1936	Cut off Sept. 12 at late yellow-ripe stage	Early yellow-ripe stage	Milk-ripe stage	Early milk-ripe stage
Date vernalized (1937)	From May 20 to May 29			
Date sown (1937)	May 29			
Date sprouts appeared	June 4			
Development of plants on:				
July 23, 1937	Tillering stage	Tillering stage	All at early shooting stage	11—shooting stage, 2—at early shooting stage
August 7, 1937	Tillering stage	Tillering stage	14—at shooting stage; 2—at early shooting stage	Heading began Aug. 2, ended Aug. 11
August 22, 1937	Tillering stage	Tillering stage, a few at early shooting stage	Heading began Aug. 10, ended Aug. 26	All headed
Sept. 1, 1937	Tillering stage	3—at shooting stage, 21—at early shooting stage	All headed	All headed
Sept. 26, 1937	18—at tillering stage; 1—at early shooting stage	4—headed, 18—at shooting stage, 2—at early shooting stage	All headed	All headed

of the other families had headed. This enables us to find out the cause of the difference in the behaviour of the families. It became clear that Families 7342, 7343 and 7346 were of a less pronounced winter habit than Family 7344. The 1936 data characterizing the behaviour of the initial plants point to the same fact.

It proved impossible to arrange the experiment with different variants of each plant, since in 1937 a comparatively small amount of seed was available. The data obtained in 1936 were insufficient to accurately establish the duration of additional vernalization needed for the grain of each separate plant. The grain from all plants was therefore given vernalization for the same period, viz. 10 days in spring of 1937. As a result of this ten-day vernalization, in the families characterized by a less pronounced winter habit the differences arising from unequal responsiveness of the seed to low temperature during the period of ripening were somewhat «smoothed over». Not only the grain which by September 21, 1936, had attained the milk-ripe stage, but also that which at that date had already reached early yellow-ripe stage was fully vernalized. This «smoothing-over» of differences was also contri-

buted to the low temperature which prevailed after the planting in 1937. Needless to explain why such «smoothing-over» did not occur in the family with a more pronounced winter habit (№ 7344).

The plants derived from seed of the same head were found to differ. We believe this phenomenon to bear above all a modificatory character. The most probable causes are:

1. Uneven development of the grain in one and the same head as a result of asynchronous pollination. Such differences when subject to the effect of low temperature are maintained for a long period of time, and result in the grains of the same head being unequally vernalized.

2. Unequal access of moisture to the different grains in the same head (especially in case of mechanical injuries). Prematurely desiccated grains are in a less favourable state for vernalization during ripening than normally developing grains.

There is but slight probability for the segregation to occur in the case considered, though such a possibility is not excluded at all.

Our experinental data thus show that developing grain is vernalized the more intensively, the less ripe it is. What is the principal cause of this regularity? There is no doubt that the gradual loss of water by the tissues of the embryo is not the chief cause. Otherwise we would not observe such a big difference between the rates of vernalization of grain at the early yellow-ripe stage and that at the early milk-ripe stage, since in both cases the grain possesses sufficient moisture for vernalization to proceed normally. We consider lacking evidence Kostyuchenko and Zarubailo's (1) assumption that vernalization during ripening takes place «within the period from the moment of embryo formation to the ripening of the grain». The data of our experiment point to the ripening grain being vernalized exactly at the time of embryo formation. In the concept «formation» we likewise include growth, in the entire absence of which the embryo cannot be vernalized. From our experimental data we conclude, in agreement with Gregory and Purvis(4), that the process of vernalization during ripening is being retarded in connection with the delayed growth of the embryo (as the latter approaches full formation). A fully formed embryo cannot be vernalized. Only after the embryo of a ripe grain starts growth anew does it regain its ability to be vernalized. In this respect we do not see any essential difference between the vernalization of ripening and ripe grain.

Grain may germinate while still on the mother plants, but the extent to which this occurs varies in different varieties. In the Spanish form of winter wheat *T. vulgare albidum*, which we used in these experiments, we failed to find any grains germinating while in the head. Other varieties, however, in rainy seasons germinate readily. Kostyuchenko and Zarubailo(1) have found that «incompletely ripened and even fully formed grain» germinates by far more rapidly than that which has attained full ripeness. In such cases a stage at which the embryo loses its ability to be vernalized is out of question.

From the fact that vernalized seeds of winter rye after having been kept for a long time in a dry state produce plants which fail to head, Gregory and Purvis(4) have drawn the conclusion that vernalization is a reversible process. In our opinion, this conclusion requires careful experimental verification. The fact that vernalization during ripening produces an effect even after the grain had been kept for quite a long time in a dry state, is a most serious proof against this conclusion. The failure of plants to head may not depend solely on the vernalization stage.

At the present time we are conducting an experiment to establish the precise stages of development of the embryo at which the process of vernalization during ripening is beginning and stopping. As material for this experiment we are using pure lines of self-pollinating plants. Gregory and Purvis[4], referring to some of their unpublished data, assert that rye embryos consisting of only eight cells are already capable of being vernalized.

In conclusion we wish to note that in any study of inheritance with respect to the vegetative period it is necessary to take into account the possibility of vernalization of grain during ripening. Types with a less pronounced winter habit segregating out in F_2 and subsequent generations may have been vernalized while still on the mother plants, and then the ratio of the phenotypes will not correspond to the actual segregation.

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PHYTOPATHOLOGY

EFFECT OF TAU-SAGHYZ SEED TREATMENT WITH THE PREPARATION «GRANOSAN»

By A. N. KLECHETOV

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Whatever point of view may be assumed when approaching the problem of the diseases of tau-saghyz seed, every observation and experiment point to a severe infection of the bulk of seed samples.

While at the outset of our work we studied the general infection of tau-saghyz sowing material, now, proceeding to a phytopathological examination, we discriminate between the infection of fruit (achaene) and that of the seed.

The symptoms of an indubitable infection of tau-saghyz seed are as follows:

1. Abundant growth of fungial colonies (*Cladosporium*, *Alternaria*, *Fusarium*, *Epicoccum*, *Torula*, *Brachysporium*, *Penicillium*, *Aspergillus*, etc.) observed when growing on the majority of fruit samples.

2. After the achaene are distributed in sterilized nutritive media, the predominant microflora consists of bacteria.

3. Many of the achaene samples exhibit an exuberant growth of fungi (*Cladosporium*, *Alternaria*, etc.) after having been treated with concentrated sulphuric acid (1.84) during 30 sec.—1 min. and thoroughly washed under aseptic conditions. In this case we have to do not only with a superficial sporulation, but with fungial development inside the fruit.

A treatment of small pieces of mycellium and conidium from pure cultures of the above-mentioned fungi with concentrated H_2SO_4 for 36—60 sec. results in their destruction.

4. Microscopic examination of achaenes reveals in some cases on the inner side of the coat of fruit or seed the presence of mycellium.

5. Seeds extracted out of achaenes previously treated with concentrated sulphuric acid, when placed under aseptic conditions in sterile moist cameras, give growth to fungial colonies. The ratio of seeds affected by fungi is lower than that of infected achaenes.

6. When seeds are grown in sterile Petri dishes, a considerable number of seedlings nearly always display obvious symptoms of infection: dying off of the entire plant, rotting of the subcotyledonous part, decay of the root, decay or shrivelling of the cotyledons, spots on the latter as well as on the subcotyledonous part.

7. Being investigated anatomically, live seeds at early stage of germination display the presence of mycellium on the inner side of the seed coat and occasionally within the embryo tissues. Alongside of colourless mycellium,

smoke- coloured mycellium is found in the tissues of living germinated seeds. This points to the fact that pathogenic forms include the so-called black fungi, which should be classed among harmless saprophites not without qualification.

8. Preliminary tests involving artificial inoculation of seeds made under laboratory conditions or after the vegetation method showed certain «black fungi» (1) to be decidedly pernicious.

The above data point to the necessity of disinfecting tau-saghyz seed.

According to the data of the field experiments conducted in 1934—1935 in Middle Asia (Atabaevo) and in the Ukraine (Oustimovka), the treatment of tau-saghyz seed with the preparation «AB» reduces the ratio of perishing plants, increasing their number on the plantation. The treatment with the «AB» preparation must therefore be practised until a better remedy be found. It is, however, necessary to search for a more effective preparation, since «AB» is not efficacious enough in the case of tau-saghyz.

Up to the present time the best results have been obtained by the Phytopathological laboratory of the Rubber Institute with the preparation «granosan» manufactured by the Institute of Fertilizers and Insectofungicides (2).

I. Laboratory Tests

1. Experiments in Germinators. When used for laboratory tests envolving growing of tau-saghyz seeds on filter paper, granosan almost totally inhibited the growth of fungi, without affecting the rate of seed germination.

Treatment with granosan (0.5—1% to seed weight) markedly reduces the infection of seeds (especially in samples with germination exceeding 60%), and reduces the ratio of tau-saghyz seedlings affected by fungi.

Table 1

Seed origin. Treatment material. Date of experiment	Dose in % to seed weight	Number of seeds treated	Energy of germination in %	Germination in %	Infection in % to germ. seeds	Infection of fruit. Amount of fungi in % to total seed amount
Seeds collected in 1934 . .	0.0	400	57.0	62.5	15.6	98.5
	0.25	400	54.5	63.75	8.62	27.25
Granosan, 27.IX. 1935 . .	0.5	400	44.25	63.25	3.95	10.0
	1.0	400	49.5	67.25	3.29	0.75
Granosan, 22.XI. 1935 . .	0.0	400	51.0	61.0	18.5	98.0
	1.0	400	37.5	58.5	3.47	0.5
Preparation «AB»	0.0	400	62.75	73.0	8.42	93.0
	3.0	400	56.0	70.0	10.00	76.75
Seeds collected in 1935, at Koynar-Bastau, granosan, 22.X. 1935	0.0	400	15.5	19.0	50.0	99.25
	1.0	400	12.5	17.75	19.72	32.25
Seeds collected in 1935 at Aksrombe, granosan, 22.X. 1935	0.0	400	22.25	29.0	19.76	99.0
	1.0	400	16.5	25.0	11.83	35.5

2. Experiment in Soil. Planting of tau-saghyz seeds treated with granosan (100 seeds per pot, each experiment involving 5 series) in sterilized and non-sterilized soil gave positive results in both cases (Table 2). The increment of plants obtained on sterilized soil was equal to 58%, that on non-sterilized—to 21%.

Table 2

Variant of experiment	Number of seeds per experiment	Number of seedlings		Number of harvested plants				Increment of plant number due to treatment in %
		in abs. figures	in % to seed weight	total in abs. fig.	average number per pot	% to seedlings	% to control taken for 100	
Sterilized soil, non-treated seeds	500	41	8.2	38	7.6±2.12	92.5	100	—
Sterilized soil, seeds treated with granosan, 1% to seed weight	500	64	12.8	60	12 ±1.95	94.0	158	+58
Non-sterilized soil, non-treated seeds	500	90	18.0	70	14 ±2.28	78.0	100	—
Non-sterilized soil, seeds treated with granosan, 1% to seed weight . . .	500	95	19.0	85	17 ±2.78	89.5	121	+21

The germination ratio of tau-saghyz fruit (and not sprouts) planted in sterilized soil is always lower than that in non-sterilized, since weakened seeds fail to germinate in the former case.

II. Field Experiments

Field experiments consisting in the treatment of tau-saghyz seed with granosan were conducted in 1937 on plots 200 m² large, each experiment involving 4 series, at 3 localities: Atabaevo (Middle Asia), Kaunchi (Middle Asia), Skadovsk (Odessa district). The results are given in Table 3.

Table 3

Average Number of Plants per Recorded Plot (Stand)

Variant of experiment	Dose of material treated with in % to seed weight	Atabaevo				Kaunchi				Skadovsk	
		Spring record		Autumn record		Spring record		Autumn record		Autumn record	
		in abs. fig.	in rel. fig.	in abs. fig.	in rel. fig.	in abs. fig.	in rel. fig.	in abs. fig.	in rel. fig.	in abs. fig.	in rel. fig.
Autumn planting											
Non-treated controls	0	634	100	490	100	1781	100	1075	100	3367	100
Granosan	0.5	1058	167	841	171	2057	116	1227	114	3590	107
»	1.0	663	105	515	105	2088	117	1480	133	3422	102
«AB» preparation . .	3.0	767	121	619	126	1926	108	1107	102	3849	114
Spring planting											
Non-treated controls	0	1425	100	1012	100	No planting in spring					
Granosan	1.0	1088	132	1227	121	—	—	—	—	—	—

In all these localities the treatment of tau-saghyz seeds with granosan (0.5—1% to seed weight) gave positive results. The increment in the number of plants recorded in spring (30—40 days after appearance of sprouts) amounted to 67%, while the increment recorded in autumn was as high as 72%.

Perishing of tau-saghyz plants during the summer season of the first year of vegetation is due chiefly to injurious insects.

At Atabaevo (autumn planting) the difference between treated seed and non-treated control exceeded the probable error by 5.2 times, thus, the significance of this difference is mathematically proven:

$$P = \frac{M_2 - M_1}{\sqrt{m_2^2 + m_1^2}} = \frac{424}{81.5} = 5.2.$$

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ZOOLOGY

ON THE RÔLE OF METAMORPHOSIS IN THE ZONAL DISTRIBUTION OF INSECTS

By I. V. KOZHANTCHIKOV

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Recent experimental investigations dealing with the influence of ecological factors upon the development of insects permit of approaching the problem of conditions which account for their distribution. One of those conditions consists in the adaptability of certain groups of insects to highly variable biotops, especially to those characterized by a short summer period. In this connection the question as to the part played by metamorphosis in zonal distribution of insects becomes of special importance.

Literature bearing on the subject is rather scarce; it is necessary to quote Handlirsch(1) who made an attempt to connect the formation of the main groups of *Holometabola* with climatic changes of the Permian. He advances the statement that the *Hemimetabola* of nowadays are more thermophilic than *Holometabola*. Contrary to this, Kennedy(2) attributes to holometabolic insects both higher metabolic activity and thermophily, supposing them to inhabit tropical and warm countries and to appear during the hot period of the year and at the warmest hours of the day.

Metamorphosis in insects may be regarded as a change in the cycle of development which is characterized by the subdivision of the latter into separate periods, each of them being characterized by a sharp prevalence of a given physiological function. Larval phases in *Holometabola* are characterized by a strong prevalence of assimilation, on one hand, and by the reduction, or in some forms even by a total lack of the trophic function during the adult stage (sex phase). Naturally enough, such a physiological specialization of different stages must be connected not only with habitus or mode of life, but also with the conditions under which some of those specific processes are developing(3). As a matter of fact, in holometabolic insects a variation of thermal optima at different developmental stages has been discovered (see the table).

From the table it may be seen that in *Orthoptera* the thermic optima do not vary for different stages, while in *Holometabola* they are observed to be somewhat different according to the stage.

The optima of humidity prove to be rather similar. Thus, in *Orthoptera* there exists after Hamilton no difference between the optima in larvae and in adults, optimal conditions in *Locusta migratoria* and *Schistocerca gragaria* being 70—80% and 50—70% of relative humidity respectively. On the cont-

Thermal Optima of Different Phases in Some Insects

Species	Thermal optimum of development in				Author, year
	egg	larva	pupa	adult	
Locusta migratoria . . .	—	32.2	—	32.2	Hamilton, 1936
Schistocerca gragaria . .	—	32.2	—	32.2	
Phaedon cochleariae . .	25—28	22—25	22—25	22	Kozhantchikov, 1937
Gastroidea viridula . .	25—28	16—25	20—22	25	
Loxostege sticticalis . .	28	32	28	25	Kozhantchikov, 1934
Agrotis segetum	25	22	19	15—20	
Pyrausta nubilalis . . .	22—30	28—30	20—28	23—28	Kozhantchikov, 1936
Panthea coenobita . . .	21.5	22.5	—	—	
Phytometra gamma . .	15—30	20—30	25	—	Kozhantchikov, 1937
Lymanthria monacha .	15	20	26	26	Zwölfer, 1934

rary, in *Loxostege sticticalis* is observed a marked difference of optima of humidity according to the phases. Thus, for praepupae it is lying at 50—60%, while in active stages it amounts to 80—90%. The same optimal distribution has been reported for *Pyrausta nubilalis*, *Agrotis segetum*, *Panolis flammea*, and some other *Holometabola* investigated.

From the comparison of the habitats of different zones the following conclusion may be drawn. Except humid tropics where climatic conditions are liable to no great seasonal changes, in southern zones of the holarctic region (steppe and forest zones) a long warm period is observed; rather considerable variability of climatic conditions is compensated there by the specifity of microclimatic conditions of the biotops inhabited by insects (soil, forest habitat, fresh water basins). In the alpine and boreal zone the seasonal variability of biotic conditions increases in connection with the reduction of the warm period and limitation of habitats (forests, soils), only cold water basins being available throughout the year.

According to the above data, *Holometabola* should be expected to be prevalent in alpine and arctic zones. In warm zones of holarctic regions *Orthoptera* and *Rhynchota* are abundant, especially as to the number of individuals; *Coccidae* are of common occurrence in the mediterranean zone. In the alpine and arctic zone *Holometabola* are prevalent, *Diptera* being very numerous too. According to the lists by Römer and Schaudinn(6), only 9 species of *Orthoptera* and 31 of *Rhynchota* are known in the arctic zone; *Coccidae* are represented only by a few forms (*Newsteadia*, *Orthesia*). Contrariwise, 786 species of *Hymenoptera*, 702 of *Lepidoptera*, 627 of *Coleoptera* and 325 of *Diptera* are known there. It is noteworthy that species of *Hemimetabola* are rather abundant in the arctic zone, different phases developing in different biotops—*Plecoptera*, *Ephemeroptera* and *Odonata*, 59, 53 and 19 species respectively.

The arctic and alpine zones are rich in *Collembola*; the latter belong in the small number of insects inhabiting the high latitudes of the antarctic zone too. The study of the ecology of alpine and boreal species of *Collembola* [Steinböck(7), Melnitchenko(5)] has shown their high hygrophily, low optima of development (0°C) and thermal limits of activity (—6°,—7°). The biotops of these species of *Collembola*, which feed on lower plants and detrit, consist in the upper layer of the soil. In alpine and arctic zones, under the snow cover, a thermic regime of about 0—2° is being maintained which permits not only of the survival, but also of the development of those species.

The study of biological processes of insects in the alpine and arctic zones reveals a high physiological specialization of different developmental stages.

According to Steinböck[8], in the glacier zone of the Alpes the development and hibernation of larvae of *Brachydiamesa*, species *Simulium*, of some species of *Trichoptera* (*Drusus*, *Stenophylax*) was observed in water basins and in glacier streams at a temperature not above 1—3°C; a short adult stage sets in during the warm period. The same differentiation of the life cycle is observed in some aquatic *Hemimetabola*, *Ephemetoptera* (*Rhitrogena*), and *Odonata* (*Dictyogenus*) too.

A life cycle characterized by a high physiological differentiation is also observed in species inhabiting desert or sand-dune habitats; some of them exhibit a great reduction of trophic organs in the adult stage. Seasonal distribution of insects offers many examples of a very high physiological differentiation of phases in the so-called early-spring and late-autumn species. Thus, the thermal optimum of the larval development in *Operophthera brumata* is as high as 25—28°C (June—July under natural conditions), while the optimum for the life and egg-lying of adults falls down to 3—4° C. The reduction of trophic organs in adults, as well as the micropterism, seems to be also accounted for by a marked physiological specialization of the life-cycle in adults (*Biston*, *Erannis*, *Operophthera*, *Malacodea*).

Concluding this article, it should be noted that a detailed study of the developmental cycles and physiological particularities of insects would surely throw light on the nature of distribution of different groups of them; up to the present time ecological and physiological data have been but insufficiently made use of in entomogeography.

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ZOOLOGY

MILK OF LARGE-SIZED CETACEANS

By B. A. ZENKOVICH

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Whilst working with the First Soviet Whale-Boat Flotilla, I took an interest in the chemical composition of whale milk. As is well known, the young of the whale at birth are nearly half the length of the cow whale and feed on the mother's milk for a little over six months; they nearly double their original length during this period of lactation and reach full maturity upon attaining the age of two years. No doubt, this rapid growth is principally due to the effect of the mother's milk.

During the whole hunt, a few lactating female whales were caught; some of them were brought to the «Aleut», a boat which served as a base, within not more than 24 hours after the catch, that is, before the carcass begins to deteriorate.

Availing ourselves of the relatively well equipped laboratory of the «Aleut» with its staff of specialists and chemists (N. R. Boyev in 1932—33 and V. B. Andrianov in 1936), we took analyses of samples of milk of five varieties of whales. In sampling the following procedure was employed: 1) pressure was applied either to the udder or to the reservoir itself for which purpose a cut was made near the lacteal glands; 2) before taking the first sample, about one litre of milk was poured out; 3) the second sample was taken after a quantity of milk exceeding 2.7 gallons was poured out; 4) the quantity of milk in the samples never exceeded 400 g and for the most part was below 400 g; 5) we took precaution that the sample milk contained no extraneous matter, such as blood, fat, etc.; therefore samples were taken only from those whales which were hit by the harpoon in parts of the body other than those containing the lacteal glands; 6) the analyses were made for fat, water and dry residue; 7) where analyses were taken only from one sample, at least 1.35 gallons of milk was poured out before sampling.

In this manner analyses of milk were taken from the five principal varieties of whales of most frequent occurrence in the World ocean: 1) two samples from the fin-whale (*Balaenoptera physalus* Lin.); 2) two samples from the blue whale (*Balaenoptera musculus* Lin.); 3) one sample from the humpback or long flippered whale (*Megaptera nodosa* Bon.); 4) one sample from the grey Californian whale (*Rhachianectes glaucus* Cope); 5) two samples from the sperm-whale (*Physeter catodon* Lin.).

The results of the analyses are tabulated below with special reference to the number of hours that passed from the moment the lactating animal was killed.

Analyses of Whale Milk

Species / Results of analysis	Fin-whale № 35		Blue whale № 186		Humpback whale № 184	Grey whale № 232	Sperm-whale № 18	
Fat (%)	44.4	42.0	45.9	34.6	38.48	53.04	38.2	36.32
Water (%)	42.0	46.5	42.6	50.5	47.48	40.58	55.25	54.22
Dry residue (%)	13.6	11.5	12.5	14.9	14.04	6.38	6.55	9.46
Number of hours that passed from the moment the animal was killed	9	9	20	20	12	4	7	7
Date when animal was killed	10.VII.1933		14.X.1933		16.VII. 1936	15.VIII 1936	6.XI.1932	

From the table it may be seen that the greatest percentage of fat is contained in the milk of the grey Californian whale, viz. 53.04%. The milk of the fin-whale and blue whale comes next for its richness in fat; on the average the milk of the blue whale contains 45.3% of fat and that of the fin-whale 43.2%. The milk of the humpback whale contains 38.48% of fat. The milk of the sperm-whale contains the lowest percentage of fat showing an average of 37.26%.

For the sake of comparison it is worth while recalling the fact that cow's milk contains 3.7% of fat, 87.3% of water and 21.7% of dry residue.

A few words about certain physical properties of whale milk: 1) when heated to 50° it forms a jelly-like mass; 2) when heated to 90° it hardens and gives out drops of fat resembling water drops; 3) the taste of the milk from all the whales (except that of the sperm-whale which I have not tasted) reminds one of Holland evaporated milk, it can be taken with coffee or cocoa instead of cream, only it has a peculiar odour to which one must get accustomed; 5) there is no fish-like smell about it; it is a peculiar «whale» smell which cannot be compared with any other smell and is not at all disagreeable.

I wish to draw attention to the fact that in 1933 and in 1936 investigations were made in the laboratory of the «Aleut» on the content of fat in nearly mature foetus of the whale. These investigations established the fact that the foetus of the whale has no blubber. From this the conclusion is to be drawn that the young of the whale cannot be born in the cold waters of the north, since being unprotected by blubber they would inevitably perish from cold. It therefore follows that whales give birth to their young in the warm tropical or sub-tropical waters. There the young of the whale have every opportunity of accumulating a sufficient quantity of blubber before migrating to the north. The accumulation of blubber must proceed at a rapid rate and that is why the milk of whales is so rich in fat.

From the table it may be seen that the blue whales, grey whales and the fin-whales, which together with their young start early on their migration to the ice-bound Artic waters, show a greater content of fat in the milk (43.2% and 53.04%); on the other hand, the varieties of whales such as the humpback which prefer warmer waters and whose migration to the north with their young is being delayed, or those like the sperm-whale which wander with their young in the waters of the warm and temperate zone, show a lower content of fat (37.26—38.48%).

The young of the whale, as is known, is unable to suck, since it has no lips while its tongue cannot function as a sucking mechanism; the milk has therefore to be squirted deep into the throat of the young which can get hold

of the nipple only with its jaws. The milk is squirted by means of a special muscle which contracts the lacteal reservoir.

I saw myself how a suckling female of the fin-whale was killed. When hit by the harpoon she slowly turned with her belly upwards and milk was running in two strong pulsating jets from both her nipples; the whale has two nipples and two udders. The jet of milk pierced the water through a distance of at least 2 m and then was slowly diffusing. The flow of the milk from one of the nipples was stronger than the flow from the other and lasted a longer time. Obviously the milk is squirted into the throat of the young with an equal, if not greater force.

Conclusions

1. The young of the whale are born in warm waters since otherwise the young which have no blubber at birth would die of cold, if born in the waters of the north.

2. The high content of fat in whale milk enables the young calf to develop blubber in a very short time before the whales start on their migration in the northern waters where there is an abundance of food.

3. The high content of fat and, probably, also of vitamins helps the young to develop rapidly, and in the course of the first year of their life to attain a size which is twice as large as that they have at birth.

4. Whales, such as the blue whale, fin-whales, and the grey whale, which start early on their migration with their young to the north and spend a certain period at the border of the ice-bound waters, sometimes passing even beyond that border, show a higher percentage of fat in their milk; whales, such as the humpback, which start on their migration with their young to the north at a later period or whales which wander in waters of the warm and temperate zone, as for instance the sperm-whale, show a considerably lower content of fat in their milk.

In conclusion I wish to remark that it would by highly interesting to take a full analysis of whale mikl, particularly for vitamins. Unfortunately, the laboratory of the «Aleut» offered no opportunities for taking such an analysis.

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ENTWICKLUNGSMECHANIK

DIE HEMMUNG UND WIEDERHERSTELLUNG DES REGENERATIONSPROZESSES DER EXTREMITÄT BEIM AXOLOTL

Von I. I. MOROSOW

(Vorgelegt von I. I. Schmalhausen, Mitglied der Akademie, d.8.V.1938)

Bei Entfernung des Epithels von den verschiedenen Stadien der Regenerationsanlagen des Axolotls entwickelt das Regenerat sich organologisch nicht, sondern bleibt auf demselben Entwicklungsstadium stehen; dagegen schreitet histologisch der Differenzierungsprozess bis zu einem gewissen Grade fort(1).

Wenn man den Regenerationsprozess nach dieser Methode hemmt und ihn darauf nach einem anderen Verfahren wiederherstellt, so kann man untersuchen, in welchem gegenseitigen Verhältnis die organologische und histologische Differenzierung im Wiederherstellungsprozess der Regeneration stehen.

1. Methodik

Die Hemmung der Regeneration wurde nach der Methode von Poležajew(1) durchgeführt. Es wurde beim Axolotl die Hinterextremität im distalen Teil des Unterschenkels amputiert. In verschiedenen Regenerationsstadien (Knospe, Kegel, Schaufelchen, Differenzierung der Fingeranlagen) wurde von der Extremität die Haut und von der Regenerationsanlage das Epithel entfernt.

Alsdann wurde die Extremität unter die vorher in der Richtung von der Hinterextremität zum Schwanz abpräparierte Haut eingenäht. Nach Verlauf von 10—12 Tagen wurde eine zweite Operation ausgeführt. Ueber dem distalen Teil der Extremität wurde ein Schnitt in der Haut gemacht und durch diesen die Regenerationsanlage hinausgeführt, die danach mit dem Epithel des Hautrandes bedeckt wurde, wodurch die Regenerationsbedingungen wiederhergestellt wurden.

Das Experiment wurde an Axolotlen (*Amblystoma mexicanum*) der pigmentierten und unpigmentierten Rasse im Alter von 10—18 Monaten in der Zeit von März 1937 bis Februar 1938 ausgeführt.

Die Ergebnisse des Experiments wurden in zwei Gruppen: typischer und atypischer Regenerate geteilt. Als typisch wurden die Regenerate betrachtet, die eine normale Struktur des Fusses und fünf oder vier normale Zehen hatten. Als atypisch betrachteten wir diejenigen, die keine normale Struktur des Fusses und nur 1—3 Zehen oder unterentwickelte, missgebildete Zehen hatten.

II. Experimenteller Teil

1. Hemmung und Wiederherstellung der Regeneration im Knospenstadium. Als Knospenstadium wurde dasjenige betrachtet, in dem das Regenerat der Extremität die Form einer gewölbten und histologisch nichtdifferenzierten Knospe besass.

Die Hemmung des Regenerationsprozesses wurde nach der obenangegebenen Methode ausgeführt, wobei sowohl in dieser, als auch in allen übrigen Gruppen eindeutige Resultate erhalten wurden: die des Epithels beraubten Regenerationsanlagen entwickelten sich organologisch nicht und blieben auf dem früheren Entwicklungsstadium stehen. Alsdann wurden mittels des obenangegebenen Verfahrens (s. Methodik) die Verbindungen für den Verlauf des Regenerationsprozesses wiederhergestellt. Zehn Tage nach der Hemmung und 220 Tage nach Wiederherstellung der Regenerationsbedingungen hatten sich in 6 von 10 Fällen (60%) typische und in 4 Fällen (40%) atypische Regenerate gebildet. Nach 60 Tagen Hemmung und 170 Tage nach Wiederherstellung erwies sich, dass sich in 2 von 6 Fällen (33.3%) typische und in 4 Fällen (66.7%) atypische Regenerate gebildet hatten.

2. Hemmung und Wiederherstellung der Regeneration im Kegelstadium. Als Kegelstadium wurde dasjenige betrachtet, in dem das Regenerat die Form eines Kegels hatte, innerhalb dessen die Skelettanlagen sich zu differenzieren begannen.

Nach 10 Tagen Hemmung und 151 Tage nach Wiederherstellung der Regenerationsbedingungen hatten sich in 12 von 14 Fällen (85.7%) typische und in 2 Fällen (14.3%) atypische Regenerate gebildet. Nach 30 Tagen Hemmung und 149 Tage nach Wiederherstellung der Regenerationsbedingungen hatten sich in 4 von 10 Fällen (40%) typische und in 6 Fällen (60%) atypische Regenerate gebildet. Nach 40—62 Tagen Hemmung und 81—222 Tage nach Wiederherstellung der Regenerationsbedingungen hatten sich in allen 15 Fällen (100%) atypische Regenerate gebildet.

3. Hemmung und Wiederherstellung der Regeneration im Schaufelchenstadium. Als Schaufelchenstadium wurde dasjenige betrachtet, in dem das Regenerat der Extremität die Form eines Schaufelchens hatte und innerhalb dessen eine weitere Differenzierung der Skelettanlagen erfolgte.

Nach 10 Tagen Hemmung und 213 Tage nach Wiederherstellung der Regenerationsbedingungen hatte sich in 1 Falle ein typisches und in einem anderen Falle ein atypisches Regenerat gebildet. Nach 40—60 Tagen Hemmung und 115—165 Tage nach Wiederherstellung der Regenerationsbedingungen hatte sich in 1 von 9 Fällen (11.1%) ein typisches Regenerat gebildet, in 8 Fällen (88.9%) wurde dagegen die Bildung von atypischen Regeneraten festgestellt.

4. Hemmung und Wiederherstellung der Regeneration im Anlagestadium mit sich differenzierenden Zehenanlagen. In diesem Stadium waren die Zehenanlagen in Gestalt von 4 Zähnchen.

Nach 10 Tagen Hemmung und 153 Tage nach Wiederherstellung der Regenerationsbedingungen hatten sich in allen 19 Fällen (100%) typische Regenerate gebildet. Nach 40 Tagen Hemmung und 250 Tage nach Wiederherstellung der Regenerationsbedingungen waren in allen 4 Fällen (100%) atypische Regenerate entstanden.

5. Kontrollgruppe 1. Beim Axolotl wurde gleichzeitig mit der Versuchsextremität auch eine Kontrollextremität amputiert. Während die Versuchsextremität alsdann den weiteren Prozeduren (Hemmung und

Wiederherstellung der Regeneration) unterzogen wurde, regenerierte die Kontrollextremität unbehindert. In allen 3 Fällen bildeten die Kontrollextremitäten typische Regenerate.

6. Kontrollgruppe 2. Die Kontrollextremität wurde amputiert und fing an zu regenerieren. Wenn das Regenerat das Stadium erreichte, in dem das Regenerat der Versuchsextremität der Hemmung unterzogen wurde, wurde von demselben das Epithel entfernt. Gleichzeitig wurden die Regenerationsbedingungen der Versuchsextremität wiederhergestellt. Auf diese Weise bedeckte das Epithel der Hauträndер gleichzeitig die Regenerationsanlage der Versuchs- und Kontrollextremität. In allen 9 Kontrollfällen hatten sich nach 149 Tagen typische Regenerate gebildet. Von den 9 Versuchsfällen hatten sich nach 149 Tagen in 3 Fällen typische Regenerate und in 6 Fällen atypische gebildet. (Die Hemmung wurde im Kegelstadium durchgeführt und dauerte 30 Tage.)

7. Kontrollgruppe 3. In 14 Fällen wurden unter die Haut des Schwanzes verschiedene Stadien regenerierender Extremitäten gebracht, deren Regenerationsanlagen das Epithel, die Organreste dagegen die Haut bewahrt hatten. Nach 19—130 Tagen wurde die Haut zerschnitten und die Extremitäten nach aussen gefördert. In allen Fällen verlief der Regenerationsprozess weiter und es bildeten sich Regenerate mit entwickelten Zehen. Das Verbleiben unter der Haut hatte nur bewirkt, dass sich in einigen Fällen atypische Regenerate bildeten, eine Hemmung der Regeneration fand jedoch nicht statt.

III. Zusammenfassung

1. Die regenerierenden Extremitäten des Axolotls wurden unter die Schwanzhaut gebracht. Sobald von dem Regenerat das Epithel und von dem Organrest die Haut entfernt wurde, wurde der Prozess der organologischen Differenzierung unterdrückt und die Regenerate blieben auf demselben Entwick-lungsstadium. Wenn aber unter den gleichen Bedingungen das Regenerat das Epithel und der Organrest die Haut hatte, so schritt der Regenerationsprozess fort, wenngleich sich mitunter auch atypische Regenerate entwickelten.

2. Die regenerierende Extremität, deren Regenerat des Epithels und deren Organrest der Haut beraubt war, wurde unter die Schwanzhaut gebracht. Die Hemmung der Regeneration wurde in den Stadien der Knospe, des Kegels, des Schaufelchens und der Anlage mit dem Anfang der Zehendifferenzierung durchgeführt. Nach einer gewissen Zeit wurde der Regenerationsprozess durch eine Verbindung zwischen dem Regenerat und dem Epithel wiederhergestellt.

3. Nach einer kurzen Hemmung (10 Tage) und darauffolgender Wiederherstellung des Zusammenhanges zwischen der Anlage und dem Epithel entwickelten sich weitaus in der Mehrzahl der Fälle (84.4%) typische Regenerate—fünf- und vierfingerige normale Extremitäten.

4. Nach einer andauernden Hemmung (40—62 Tage) und darauffolgender Wiederherstellung des Zusammenhanges zwischen der Regenerationsanlage und dem Epithel entwickelten sich im wesentlichen (91.2%) atypische Regenerate—Extremitäten mit 1—3 Zehen und Missbildungen.

5. Im Hemmungsprozess schritt die histologische Differenzierung der Regenerationsanlagen fort(1). Die Bildung atypischer Regenerate nach der Wiederherstellung des Regenerationsprozesses erfolgt auf Kosten des Fortschreitens der histologischen Differenzierung während der Hemmung der Regeneration. Im normalen Regenerationsprozess sind die beiden Prozesse der histologischen und organologischen Differenzierung gegenseitig abgestimmt.

6. Balinsky(2) interplantierte in die Augenhöhle der Tritonlarvae einen mesodermalen Teil der Extremitätenanlage. Aus den Interplantaten entwickelten sich knospelige Skelettelemente der Extremität, deren Entwicklungsgrad geringer war, als der der Skelettelemente einer normalen Extremität.

Also besteht zwischen den Angaben von Balinsky(2) einerseits und den von Poležajew(1) und mir ermittelten andererseits eine gewisse Analogie.

Die histologische Bearbeitung des Materials und weitere experimentelle Untersuchung werden fortgesetzt.

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DEVELOPMENT MECHANICS

GROWTH AND TRANSFORMATION IN VITRO OF THE IRIDESCENT PORTION OF THE CHOROID COAT (TAPETUM)

By J. A. YINNIKOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 25.V.1938)

The author's experimental material consisted of the eyes of pig embryos, 5 to 18 cm long, and those of newborn and adult rabbits, both albinotic and pigmented. Cultures were followed up for a period of over 67 days. As is known, tapetum material at the stages of eye anlage, optic vesicle and optic cup consists of a layer of prismatic cells with nuclei lying at different levels. In its further development the tapetum lags somewhat behind the overlying perceptor membrane and becomes pigmented. At first the pigment appears in the form of granules; these later become needles and crystals.

With the eye's growth and development, the tapetum is converted into one single layer of short cylindrical elements, each cell having a large spherical nucleus in its center. These regularly shaped, mostly hexagonal cells can be traced at this stage in 13 cm pig embryos. Numerous mitoses may be seen. Mononuclear cells occur most frequently. In newborn rabbits, the tapetum nuclei are most typical, the cells containing as a rule two nuclei. The mode of proliferation of the tapetum elements in these stages has been frequently brought up in the literature, the prevailing opinion being, erroneously, in favour of amitosis([1],[2]). Binuclear cells are formed by mitosis which, in these stages, is not accompanied by plasmotomy; the latter is apparently prevented by the geometrically regular form of the cells, the majority of them being six, five, four, eight or seven-sided.

The changes occurring in the described initial material on explantation were traced both in vivo and in sections. If the tapetum was stripped off together with the overlying choroid, the latter usually lay on the inner surface, while the retinal elements prove to be on the surface of the explantate. No noticeable changes occurred during the first 24 hours.

On the second day the polygonals along the edges of the explantate tend to assume a spherical form; the nucleus swells up. The various elements pile up on each other, gradually moving into the fibrin, where they form various structures in the zone of growth. The different portions of the explantate take no equal part in the process of penetration. The transformations observed on the non-penetrating surface of the explantate are of peculiar nature. The rounded and proliferating elements overlie each other without, however, being pushed into the fibrin. The cells making up this accumulation are noted to be chiefly binuclear. Mitoses are occurring only in mononuclear

cells. In some of them vacuoles are developing. Sooner or later the non-penetrating accumulation on the surface of the primary piece reaches the fibrin, forming there an extensive zone of growth. The cells flatten out without entirely separating. The transfer of the zone of growth is not always accompanied by increased mitotic activity. Here an interesting phenomenon is observed connected with the fact that most of the cells are binuclear. Binuclear elements having moved into the zone of growth, some distance from the center of the explantate, as a rule, undergo plasmotomy and only after the protoplasmic body corresponds to a single nucleus does the cell begin to divide by mitosis.

A great variety of intricate structures arises in the zone of growth. Epithelium-like membranes may be seen to surround the bit more or less regularly. Most frequently there are several membranes each lying in a different plane. Ties are generally growing in between the membranes and are bound with them. The membranes and ties adhering and forming a thick layer may form complex growths of more intricate structure. In the zone of growth there are sometimes observed special cell complexes of bizarre stellate form, occasionally also loose net-works of pigmented binuclear elements connected by elongated delicate filaments of protoplasm. Those various structures may be observed at all stages of cultivation and are mutually interchangeable.

The pigment observed in the zone of growth is twofold in its character, as well as in time and place of formation. In intensively growing cultures close to the primary bit the pigment migrates from the maternal cells. The cells around the edges receive less and less pigment till at the periphery none can be detected. For this reason Kapel denies that pigment is produced in vitro, however, if the culture is retarded in development, which is not always due to a lack of embryonal extract, it is possible to detect pigment formation in peripheral portions previously altogether lacking pigment, at first in isolated cells, later in entire cell groups. Newly formed pigment is recognized by its granular form and rather light, yellowish colour. The young granular pigment does not, as in the organism, become converted into crystals. The pigment is formed independently of chondriosomes. No conversion of pigment into fat takes place. The characteristic fibrillation in the zone of growth of the tapetum is observed in preparations fixed in Zenker's formalin and stained with iron haematoxylin. Some cells in the zone of growth do not undergo plasmotomy.

The unclear and intricate picture of the zone of growth in the tapetum in under surface development is the result of the following factors: constant supply and active migration of new cell material, coming from the primary piece, into the fibrin, and the growth and proliferation of elements in the zone of growth itself. Besides, some portions of the zone of growth are more rapid in growth and proliferation than others. Several complexes growing simultaneously push forward in mutually perpendicular or oblique planes. For example, when the elongated cells of the ties lie radially to the center of the explantate, then the elements of the underlying membrane grow concentrically. Cells of the overlying membrane are characterized by an oblique direction of their long axis, etc. This phenomenon may sometimes be found within the limits of one membrane when there is no growth in other planes. No epithelization of the primary piece is observed, contrary to Kapel's data(3).

Analysis of the transformations of the tapetum acquires new significance when the data here obtained are compared with the facts connected with problems of the development mechanics of the eye. It should be noted that up to recently, the retinal layers of the eye have been regarded as epithelial. However, the tapetum differs from the epithelium in being a completely determined peculiar tissue which is known as such in amphibians and birds at early

stages of ontogenesis, viz. in the neurula. The morphology of the tapetum as seen in the organism and in tissue cultures allows us to place it close to the other derivatives of the eye anlage and also to several other derivatives of the neural plate. The closest degree of similarity, as could be expected a priori, was found to exist between the tapetum and the retinal layers and the myoneural elements of the edge of the optic cup(4).

Special significance may be attached to the structures developing upon the non-penetrating surface of the explantate. Up to a certain degree these changes recall those occurring in the tapetum in *Retinitis Pigmentosa* (vacuolization, loss of polygonal form, mitoses). On the other hand, there is a striking similarity between these multi-layered accumulations on the explantate surface and the histological relations known to occur at that stage of the development of tapetum in the organism when the presumptive anlage, consisting of a series of elements with nuclei located at different levels, is being converted into the optic vesicle and afterwards into the optic cup, that is, when the eye cup finally becomes histologically fully determined. The similarity is supported by the identical distribution of mitoses. Thus, in tissue cultures we note transformations which are reminiscent of early histological structures in the organism, but which follow the opposite direction.

While emphasizing the specificity of the cytotypical peculiarities of the tapetum elements, it should be noted that this portion of the optic cup has in vitro histological potencies in common with other derivatives of the neural embryo. For example, there is a definite similarity between the character and form of the palisade-like arrangements of membranes and ties occurring in tissue cultures of elements from the optic paths (5), the covering cells of the vascular plexuses of the brain(6), and the elements of the pia-arachnoidea(7). Especially striking is the similarity between the elements of the optic paths and the object under investigation. The similar range of transformations in this case may be accounted for by the fact that the future glial elements of the optic paths appear simultaneously and in connection with the presumptive eye anlage. At early stages of ontogenesis in amphibians the anlage of the optic paths may on transplantation develop into an optic cup. For this reason Adelman(8) designates the entire system of eye anlage together with the interstitial material of the future optic paths as the optico-ocular apparatus.

The similarity noted between the expression of histoblastic potencies in the tapetum tissue, on the one hand, and the retinal layers and myoneural elements of the edge of the optic cup, as well as several other derivatives of the neural embryo, differentiated along various lines, on the other hand, is due to their common origin and to a certain extent to their similar position in the organism. All these tissues are auxilliary components of the central nervous system and taken as a whole form the glial type.

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DEVELOPMENT MECHANICS

ON THE DETERMINATION OF ENTODERMAL ORGANS IN AMPHIBIA

By B. I. BALINSKY

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The determination of entodermal organs is at present investigated far less than the determination of organs, belonging to the other two germinal layers: the ectoderm and the mesoderm. The only paper dealing specially with the determination of entodermal organs in amphibia is that of Holtfreter(3). In this work Holtfreter arrived at the conclusion that the entodermal organs (liver, pancreas, intestine) are being determined as early as in the middle gastrula stage. Further investigations by the same author (4) seemed to corroborate this conclusion and allow to extend it to the early gastrula stage. Already in this stage the entoderm, according to Holtfreter's data, seemed to be a mosaic system, whereas the ectoderm and the mesoderm retain a high regulative capacity until the end of gastrulation.

Holtfreter in his work was using the method of explantation and interplantation. I deemed it desirable to investigate the determination of the entodermal organs by the same method, as was set at the base of the classical investigations on the determination of ectoderm by Spemann(9) and Mangold(6), namely by the method of transplantation. Accordingly I made a series of experiments during the spring of 1937.

As the yolk-laden entoderm of early embryos heals very badly, I transplanted pieces of entoderm, the determination of which I wanted to investigate, into the blastocell of early gastrulas or blastulas. Xenoplastical (*Triton taeniatus* ♀ *Amblystoma mexicanum*, *Tr. cristatus*→*A. mexicanum*) or heteroplastical (*Triton cristatus* → *Tr. taeniatus*) transplantation was made use of, or else, if the transplantation was homoplastical (*Tr. taeniatus*) the donor was stained beforhand with nile-blue sulphate. In this way I could ascertain the position of the graft in the living operated embryos, and in the case of xenoplastic transplantation (*Triton—Axolotl*) also to distinguish the graft and host tissues on sections by the different size of cells and nuclei (those of the *Axolotl* are perceptibly bigger). When I studied the operated embryos in sections, especially in cases of xenoplastic transplantations, it became evident that the grafted entodermal material did not perish, but fused with the entoderm of the host; as a rule, the graft was harmonically incorporated in the gut of the host and participated in the formation of its different parts in accordance with the eventual position of the graft.

The grafts were taken from embryos in different stages, namely: the early gastrula stage, the late gastrula stage and the neurula stage. The grafts consisted in pieces of the entoderm. The location, the pieces were taken from, was registered and portrayed schematically. Of over 280 operated embryos a considerable number died, in many cases the position or the origin of the graft were doubtful. In 50 cases the origin, the position and the differentiation of the graft were made out with certainty. As it was not possible to control the position of the grafts at the time of the operation, the graft often came to lie in the same region to which it appertained originally. But still more often the graft became situated in part or wholly in a strange region and it is from such cases that information on the determination of the entodermal material may be obtained.

As already mentioned, in the vast majority of cases the grafts formed organs and tissues in accordance with their new position. The grafted tissue as a rule was fully incorporated and did not disturb the structure of the gut at the site of its location. This result shows that the parts of the entodermal material are not determined or at least that their determination is very labile. Remarkably enough not only grafts taken from early and late gastrulae differentiated in accordance with their new position, but also those taken from neurulae. Of 14 successful grafts pertaining to this stage, 6 differentiated contrary to their prospective significance, but in accordance with their new position (the other 8 grafts came to lie in regions, that corresponded approximately with their original position in the donor embryo).

From these results it must be concluded that the entoderm is no mosaic system neither at the late nor early gastrula stage, nor even at the neurula stage. The parts of the entodermal primordium are not strictly determined to form certain entodermal organs. In one case the entodermal graft seems to have differentiated into cartilages of the branchial type, thus transgressing the limits of its germ layer. The self-differentiation of the entodermal parts in Holtfreter's experiment obviously proceeded not on the basis of stable determination, and very possibly, did not correspond to the prospective significance of these parts [comp. the «bedeutungsfremde Selbstdifferenzierung» in the experiments of Kusche(5) and Bautzmann(2)].

The question has been as yet left unanswered, whether the entoderm in the investigated stages lacks any kind of determination, or whether its determination is readily reversible. One case in my experiments in which the graft retained its original line of differentiation seems to furnish evidence in favour of the second alternative. In this case the graft was taken from the late gastrula stage and normally should have formed the most anterior part of the gut. After transplantation it came to lie in the trunc region, but unlike the other cases it was not incorporated in the host gut, but formed a vesicle lined with epithelium which is very similar to the epithelium of the mouth or farynx. The fact that the graft did not heal into the wall of the host gut seems to have given the graft an opportunity to retain and develop further its original labile determination that becomes lost in other grafts, when they fuse more closely with the host entoderm. A labile determination would also more easily explain Holtfreter's data. There is, however, no reason to suppose that the organs and differentiations of the entoderm are marked out somehow, if not finally, at these early stages. The observed facts could be a result of the existence of some gradual differences (a gradient?!) determining the regional character of the entodermal material.

Be this as it may, it is obvious for us that the entoderm is determined very gradually, not earlier, but later than the ectoderm and the mesoderm (which would correspond with the late differentiation of parts of the gut). The fate of different parts of the entoderm is not determined by mosaic fac-

tors of self-differentiation, but an interaction of parts is an essential factor in this development. When the entodermal tissues become finally determined, and whether such a determination ever takes place—remains to be shown by further experiments.

It may be pointed out that recently the conclusion as to the plasticity of the entodermal primordium has been reached by investigators of the avian development(7), so that this conclusion will probably hold true for all classes of vertebrates.

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PALEONTOLOGY

CORALS *RUGOSA* FROM THE MIDDLE AND UPPER THE CARBONIFEROUS DEPOSITS OF THE DONETZ BASIN

By V. FOMITCHEV

(Communicated by A. A. Borissiak, Member of the Academy, 23.V.1938)

Rugosa corals have been described from the Donetz basin by N. Yakovlev(5) (dolomite series) and by K. Lissitzin(4) (Lower Carboniferous).

In 1935 the author started a detailed study of *Rugosa* from the Middle and Upper Carboniferous of the Donetz basin, using as material the collections of the Central Geological and Prospecting Institute obtained in the Donetz basin by a group of geologists during all the period of detailed geological work in the basin. The principal material was obtained from the collections of L. Lutugin and P. Kumpan. The coral faunas were found to occur in great abundance in the Upper Carboniferous and in the upper part of the Middle Carboniferous; the lower series of the Middle Carboniferous (C_2^1, C_2^2, C_2^3 C_2^4) are poor in coral remains. The series are given here after Th. Chernyschev and L. Lutugin under the indices used in the detailed geological map of the basin (1:42000). The Middle Carboniferous was subdivised into 6 series (C_2^{1-6}), the limestones of which have the indices F, G, H, I, K, L respectively; the Upper Carboniferous is subdivided into three series (C_3^{1-3}), the limestones corresponding to M, N, O respectively.

Among the compound corals of the Donetz basin abundantly represented is the genus *Lithostrotionella* Y. et H. which flourished in the upper parts of the Middle Carboniferous, supplanting, as it were, the Lower Carboniferous *Lithostrotion*, finds of which in the Middle Carboniferous of the Donetz basin are of rare and dubious occurrence. The group of species of *Lithostrotionella* appears in the upper parts of the series C_2^5, richly develops in the series C_2^6 and dissappears in C_3^1 (maximum development in limestones L_5). The uppermost parts of the series C_2^5 and the lower parts of the series C_2^6 include another characteristic group referred by the author to the new genus *Denophyllum* (fasciculate corals differing from *Diphyphyllum* in the presence of long septa uniting in the centre), maximum development in the limestone K_8.

Genus *Cystopora* Y. et H. of the same type as is encountered in the upper parts of the Middle Carboniferous in the Moscow basin is represented in the

Donetz collection by a number of forms, the specimens being here not very numerous. It is found beginning wiht the upper parts of the series C_2^6, in C_3^1 and C_3^2. In the Moscow region this genus is richly represented in the Miachkovo beds of the Middle Carboniferous(1–3). Here are also met with representatives of the genus *Arachnastraea* Y. et H. closely allied to the species established by Yabe et Hayasaka on the base of material from the upper parts of series C_3 of Eastern Asia. In the Moscow region *Arachnastraea molli* (Stuck.) occurs in the Miachkovo beds of the Middle Carboniferous.

The *Lonsdaleidae* of the Donetz basin are of exceptional interest and it is in the Donetz basin that the presence of typical representatives of the genus *Polythecalis* Y. et H., met with in the series C_3^1, and of the genus *Lonsdaleiastraea* Gerth characteristic of C_3^3 was stated for the first time in the USSR. Both genera are regarded in Eastern Asia (China, Japan, Timor) as typical Permian ones and are not known from the Carboniferous. The Donetz forms are as highly organized forms as the Chinese and Timorian ones.

The fauna of simple corals with columella is extremely varied in the Donetz basin. The family *Lophophyllidae* besides its three principal genera (*Lophophyllum* Edw. et H., *Koninckophyllum* Thom. et Nich. and *Lophophyllidium* Grabau) is represented in the Donetz basin by a number of new genera (*Agaricophyllum*, *Yuanophylloides* and *Neokoninckophyllum*). The genus *Lophophyllum* Edw. et H. is taken by the author in a restricted sense; it differs from the genus *Koninckophyllum* Thom. et Nich. in a number of minute but important features; it is represented in the Donetz basin by three species encountered in different series of the Middle Carboniferous. All the four species of the genus *Koninckophyllum* found here are described from series C_3^1. Forms referred by the author to the new and abundantly represented genus *Neokoninckophyllum* were found in the same series. The genus *Lophophyllidium* Grabau is represented in the Donetz basin by a number of forms; it shows a maximum development in the series C_3^2; one of its species is known from series C_3^3. *L. proliferum* McChesn., described in 1903 by N. Yakovlev from the Permian dolomite series of the Donetz basin, is taken for the genotype of this genus, as it was also proposed by Fr. Heritsch. This form is redescribed by the author of this article as *L. yakovlevi* nom. nov.

Corals referred by the author to the family *Clisiophyllidae* are rather varied though not abundant quantitatively. Of the genera previously known there were identified: *Histiophyllum* Thom., *Kionophyllum* Chi, *Carcinophyllum* Nich. et Thom., *Axophyllum* Edw. et H. The genera first of these is found in the series C_3^1 and the Permo-Carboniferous series. Single specimens of *Kionophyllum* Chi and *Carcinophyllum* Nich. et Thom. were found (judging by labels) in the series C_3^3. A fragment of *Axophyllum* sp. is found in series C_2^6. The following new genera are established: *Sestrophyllum* (C_2^5, C_3^3, *PC*) and *Dibunophylloides* (C_3^1). The absence of representatives of the genus *Dibunophyllum* Thom. et Nich. is characteristic of the collection. According to the published data, typical *Dibunophyllum* are absent in the Middle and Upper Carboniferous of other regions as well, i. e. this genus may apparently be considered as a typical and leading Lower Carboniferous genus.

Simple corals without columella are also characterized by great variety and are represented by a great number of forms. The Donetz collection* inclu-

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* The bulk of the collection is very large; the material is distinguished by a perfect preservation of the internal structure of the corals. The stratigraphic horizons from which separate specimens were taken are mostly referred to the labels. At the present time the study of a considerable part of the material is achieved (compound forms and simple forms with columella).

des more than 100 species of *Rugosa*. The majority of them are new ones. The groups of corals studied are sufficient for series C_2^5, C_2^6, C_3^1, and C_3^3 to be rather fully characterized.

The series C_2^5 is characterized by a maximum development of the genus *Donophyllum* (3 forms) and peculiar colonial *Campophyllum*. The genus *Lithostrotionella is* absent here.

The series C_2^6 includes many *Lithostrotionella* (4 forms); in its lower parts *Donophyllum* disappears.

The series C_3^1 is characterized by a rather rapid disappearance of the genus *Lithostrotionella*, the genus *Cystophora* is represented by a number of forms; typical *Polythecalis* are met with. The group of simple forms with columella attains here its maximum development and is characterized by a considerable number of *Neokoninckophyllum* (2 species), *Koninckophyllum* (4 species), *Yuanophylloides*. In the Donetz basin the representatives of these genera are almost exclusively confined to the series C_3^1. A number of species of *Dibunophylloides* and *His tiophyllum* also occur in the series.

The series C_3^2 is characterized by 2 species of *Cystophora*; representatives of the genus *Arachnastraea* are met with on its boundary with the series C_3^1. A number of species of *Lophophyllidium*, absent in C_3^1, is found here.

The genus *Lonsdaleiastraea* appears in the series C_3^3, while all the other compound forms abundantly represented in the lowerseated series are unknown. Of the simple forms 2 species of *Sestrophyllum* and representatives of the genus *Kionophyllum* should be mentioned. *Lophophyllidium* of the same type as in the series C_3^2 are of rare occurrence.

The same groups as in C_3^3 (*Sestrophyllum*) continue to develop in the lower parts of the series *PC* (collections are insignificant); *Histiophyllum* is known.

All the above said proves that *Rugosa* may serve as good guide fossils for the Middle and Upper Carboniferous.

There is observed a certain general resemblance of a number of compound forms of the Donetz basin with the fauna of compound *Rugosa* from the Miachkovo horizon of the Middle Carboniferous in the Moscow region. Common forms are: *Cystophora* aff. expansa Dobrol., *Cystophora humboldti* (Stuck.), *Cystophora freislebeni* (Stuck.), *Arachnastraea molli* (Stuck.). The occurrence of these forms in the series C_3^1 (and even C_3^2) confirms the statement already known in literature that the series C_3^1 corresponds to the upper parts of the Middle Carboniferous of the Moscow region (Miachkovo). But it is so far impossible to draw a boundary (on the base of corals) between the Upper Carboniferous and the Permian in the Donetz basin since the fauna from the series overlying the series C_3^3 is still very imperfectly studied.

The finding of typical representatives of the genera *Polythecalis* and *Lonsdaleiastraea* in the series C_3^1 and C_3^3 of the Donetz basin points to the fact that one should be extremely careful when determining the age of the Upper Paleozoic beds using the literature on corals now available for Eastern Asia (China, Japan, Timor). Both these genera are regarded there as typical Permian ones and are unknown from the Carboniferous. A certain part of these limestones may possibly turn out to be of Carboniferous age.

The nearly total absence of similarity between the coral faunas of the Middle and Upper Carboniferous of the Urals and those of the Donetz basin, judging by the material published, should be noted. It may possibly be explained by the fact that the corals of the Urals have been collected from other horizons than those containing corals in the Donetz basin and Moscow region.

After the study of the whole fauna of the Middle and Upper Carboniferous will be completed, conclusions as to the affinity of these faunas may undergo some changes.

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PALEONTOLOGY

ON THE FAUNA OF TERTIARY *CRICETIDAE* OF OF THE USSR

By A. I. ARGYROPULO

(Communicated by A. A. Borissiak, Member of the Academy, 23.V.1938)

As regards the number of species and habitats it is the hamster which holds the first place among the Tertiary fauna of Europeans rodents. Fossil hamsters of Europe are known best of all, next comes America, and of late discoveries in Asia (China and Mongolia) have been reported. In the Soviet Union we have so far been familiar with nothing but remains of Recent species of Quaternary deposits, and therefore the first finds of Tertiary hamsters are of exceptional interest. Further accumulation of material is most desirable, as it would permit to base a series of zoogeographical conceptions upon real scientific data and would, perhaps, shed light on the question as to the origin of the greater part of hamster-like and mouse-like rodents.

Cricetodon (*Palaeocricetus* sbg. n.) *caucasicus* sp. n. № 42—247, M_2 inf.; № 47—245, M_1+M_2 inf. and d. incisive in the fragment of dentale; № 47—246, M_1 inf.; № 47—248, mandible (deformed) and part of maxilla with complete sets of molars (inf.—sup., left side). Ossiferous remains poorly preserved, feebly mineralized, colour light brown. Teeth in excellent state of preservation. All enumerated specimens are to be found in the collection of the Paleontological Institute of the Academy of Sciences.

H o r i z o n. Middle Miocene, right bank of the river Kuban, stanitza Byelometchetskaya, Cis-Caucasian region.

This species differs sharply from the majority of representatives of *Cricetodon* Lartet by the strongly simplified pattern of the grinding surface of the molars, and it can be as to this character classed among some European Miocene and Pliocene species: *Cr. sansaniensis* Lart., *Cr. larteti* Schaub, *Cr. lusitanicum* Schaub, *Cr. angustidens* Deparet. Of the species enumerated, *Cr. caucasicus* is in particularly close relation to *Cricetodon sansaniensis*, widespread in the Vindobonian stage of the Miocene, up to the confines of the Pontian age. In both of them the anterior cusp on M_1 inf. is very diminutive, and the union of the metaconid of this tooth with the protoconid takes place not in the anterior part, but in the posterior, or else, the metaconid does not unite in front directly with the anterior innominate cuspule, the connection taking place posteriorly through the protoconid. In *Cr. larteti*, *lusitanicum* and *angustidens* is exhibited a broad fusion of the large innominate cuspule of M_1 (inf.) directly with the metaconid. As far as the distinctions of *Cr. caucasicus* from *Cr. sansaniensis* are concerned, they may be summed up as follows: length of molars in *Cr. sansaniensis* M_{1-3}(inf.)=7.2—8.2 mm, M_{1-3}

(sup.)=6.8—8.2 mm. The measurements of the molars in *Cr. caucasicus* give the following figures: M_{1-3}(inf.)=6.1 mm, M_1 (inf.)=2.1 mm (another specimen); M_2=2 mm, M_{1-2}=4.3 mm, M_{1-3} (sup.)=6.2 mm. Thus, the Caucasian hamster is smaller than *Cr. sansaniensis*. In *Cr. sansaniensis* M_1 (inf.) is usually somewhat longer than M_2, whereas in *Cr. caucasicus* exactly the reverse is the case. By the pattern of the molars of the lower series *Cr. caucasicus* is in general indistinguishable from *Cr. sansaniensis*; as to the upper series, a more constricted anterior portion of the first molar should be noted. The following details of the structure of the molar's grinding surface may be emphasized: protoconid of M_2 very narrow, there is no trace of the mesostylid heel, cingulum posterior («Schlusscingulum» after S. Schaub) well developed, and not inferior to hypoconid in size; M_3 also exhibits a very narrow protoconid; the bifurcation of the anterior cusp of M_1 (sup.) is faintly visible, and the crest («Längsgrat»), connecting the paracone with the posterior inner cuspule, is relatively very long; M_2 has a feebly developed cingulum superior, and the protuberance of the mesostyle (the anterior branch of the posterior inner cuspule) is perfectly visible. Of all the above-mentioned characters, the most clearly pronounced are such as slenderness and small size of the protoconid, strongly developed cingulum posterior on M_2 (inf.), structure of the anterior cusp («Vorderknospe» after Schaub) on M_1 (sup.).

The lower jaw of *C. caucasicus* is distinguished from that of the Recent hamsters by a more sloping diastemal incisura and incisura on the lower edge of dentale, at the base of processus angularis. We are not in position to give any more details, as the available fragment of the lower jaw is to a considerable extent deformed.

European Tertiary hamsters present a group abounding in species, and a very mixed too, the principal mass being referred to the genus *Cricetodon* Larted. The first finds of *Cricetodon* fossils were recorded in Oligocene, the very lowest (Quercy phosphorites), and the latest, relatively not numerous, are known from the Pontian stage of the Miocene (Portugal) and Pliocene (Perpignan). In the majority of cases the remains consist of molars of both jaws, and therefore the generic characters are being inferred on the basis of the structure of the grinding surface of the molars (see further). Fragments of skulls and entire skulls are known but of a limited number of species. However, if one is guided in the descriptions not by the characters of teeth alone, one sees the distinctions between separate *Cricetodon* «species» to be often more sharply pronounced, than generic distinctions between many Recent groups of hamsters. For instance, the skull of the diminutive *Cricetodon minus* Lartet from the Vindobonian stage of Miocene shows amazing likeness to any one representative of the Recent American genus *Reithrodontomys;* others, like the Miocene *Cr. affine* Schaub., or the Oligocene *Cr. incertum* Schloss., remind one by their massive skull rather of representatives of contemporary *Oryzomys*, or even *Cricetulus*. Naturally enough, Miocene *Cr. minus*, *Cr. affine* etc. possess more primitive vestiges in common, than contemporary American or European species, yet the differentiation into separate genera, say of *Reithrodontomys* or *Oryzomys*, can be made without hesitating. The example quoted shows plainly how conventional and artificial is the uniting of the majority of Tertiary hamsters into one genus *Cricetodon* merely on the ground of the molar pattern. However, the odontological characters too are variable, and may serve as a basis for quite a precise division of *Cricetodon* into several groups. Thus, the most complex pattern of molars refers to the earlier Oligocene and partly Miocene forms, while a considerably simplified one belongs to species of the end of Miocene and Pliocene. The latter are relatively close allies of the Recent hamsters (especially, *Cricetus* and *Mesocricetus*). These are the above-mentioned Miocene *Cr.*

sansaniensis, *larteti*, *lusitanicum*, the Lower Pliocene *angustidens* and the herein described Middle Miocene *caucasicus*. For this whole group we suggest a new designation—*Palaeocricetus* (type *Cr. sansaniensis* Lartet 1851). Representatives of *Palaeocricetus* are of medium size, the tooth row (inf.-sup.) measures from 5.0 to 8.5 mm in length, «Längsgrat» is very much shorted, there is no trace of a mesostylid heel, the posterior branch of the protoconide («Protoconidhinterarm») and the anterior branch of the posterior inner cusp («Mesostylsporn») are strongly reduced or altogether lacking. All the tubercles are relatively large, bunoid, not stretched in the shape of transverse wrinkles.

It is quite possible that *Palaeocricetus* are the nearest ancestors of the Recent *Cricetus* and *Mesocricetus* [see *Cricetinae* in «The Fauna of the USSR»(4)]. Through a long chain of species, as, for instance, the Oligocene *Cr. gerandianum* Gerv., or the Miocene *Cr. breve* Sch., *bourgeosii* Sch., *affine* Sch., etc., *Palaeocricetus* is linked to the real *Cricetodon* of the Oligocene and of the beginning of Miocene.

Cricetops affinis sp. n. № 1442/94 of the collection of the Paleontological Institute of the Academy of Sciences. a) A whole tooth row of a lower series: M_1 and M_2 detached; M_3 in a small fragment of dentale (mounted in plastic clay); b) a whole tooth row of a lower series with the alveolar portion of dentale. The surface of molars somewhat more worn than in the preceding specimen (the one as well as the other are quite adult animals). All enumerated remains are in an excellent state of preservation. The jaw fragments and teeth are of very light, brownish colouring.

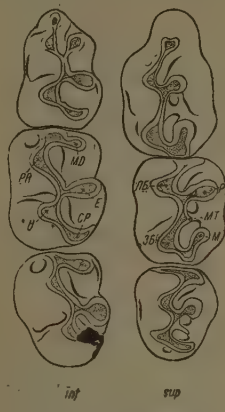

Fig. 1. Molar teeth of *Cricetodon* (*Palaeocricetus*) *caucasicus*: A. Arg. *PR*—protoconid, *H*—hypoconid, *MD*—metaconid, *E*—entoconid, *CP*—cingulum posterior, *P*—paracone, *M*—metacone, *MT*—protuberance of mesostyle, *ПБ*—anterior inner cuspule, *ЗБ*—posterior inner cuspule.

H o r i z o n: Upper Oligocene, Indricotherium layers. Myn-Say, Central Kazakhstan.

The species herein described is closely related with *Cricetops dormitor*, the only heretofore known representative of the genus described by Matthew and Granger(1) from Oligocene (Hsanda-Gol formation) of the Mongolia.

The following characters are distinguishing *Cr. affine* sp. n. from *Cr. dormitor*: M_1 hardly elongate or equal to M_2, whereas in *Cr. dormitor* the first molar is markedly longer than the second. M_3 is shorter than M_2; in the Mongolian species the reverse is the case. The reduction of the last molar is a progressive feature for all representatives of *Cricetinae*, both fossil and Recent (especially), and there is no contradiction in this case. The pattern of the grinding surface of the molars on both species *Cricetops* is very much alike. As we know practically nothing of individual variation in separate elements of molars in *Cricetops*, we are able to draw very few positive conclusions, and are therefore pointing out here but one character of M_1: the entoconid in *Cr. affinis* lies almost at the same level as the hypoconid, and the protuberance of the mesostylid on the «Längsgrat»(2) is small and therefore lacks the dentine islet.

M e a s u r e m e n t s: length of the tooth row (upon the crowns) of the lower series—9 and 9.1 mm. Length of M_1—3.2, M_2—3.1, M_3—2.8 mm. Thus, by the total length of the tooth row *Cr. affinis* is not discernible from *Cr. dormitor*.

The recovery of a species, closely related to the Mongolian, in Tertiary

deposits of the Central Kazakhstan is of great zoogeographical interest. This finding is indicative of a faunistic link, which was been in existence since a long time between Mongolia and Kazakhstan, and has not disappeared, as we are aware, even up to this date. In this connection, note should be taken of the ancient habitat of the Recent hamster *Allocricetulus eversmanni* Br., passing in a rather narrow strip through the central regions of Kazakhstan, reaching westward the Volga river, continuing eastward into Mongolia and extending there as far as the central regions. Of the modern Mongolian species penetrating far into the Northern Kazakhstan, one may indicate another hamster *Phodopus sungorus* Pall. and a jerboa *Allactaga saltator*, the latter spreading westward to the Caspian. In the Pleistocene yet Kazakhstan abounded in species of the genus *Ochotona*[3]. Nowadays they are of rare occurrence there and one of them, widespread in Mongolia (*Ochotona pricei* Thos.), spreads there westward not farther than the northern region by the lake Balkhash (the present genus *Ochotona* is considered to be native of Central Asia).

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PALEONTOLOGY

THE RECOVERY OF A TRIASSIC ANOMODONT IN THE ORENBURG PROVINCE

By I. A. EFREMOV

(Communicated by A. A. Borissiak, Member of the Academy, 27. IV. 1938)

At the end of 1936 the geologist of the Eastern Oil Trust («Vostokoneft») P. I. Klimov presented to the Paleontological Institute of the Academy of Sciences of the USSR a large concretion with a part of a skeleton of a fossil reptile. The concretion was found by P. I. Klimow in the outcrop of variegated rocks on the river Donguz, opposite the settlement Petrovskoye, 25 km east of the station Mayachnaya, Tashkent railway.

After the development it appeared that the concretion contained an incomplete skeleton with a skull of one of the representatives of the family *Dicynodontidae* of the order *Anomodontia* (collection 159/1 PIN).

The skull of the skeleton 159/1 displays a number of characters distinguishing it from the genus *Dicynodon*. Thus, the intraorbital portion of the skull roof is wide, its anterior angles being formed by praefrontalia, extending into processes overhanging the orbits. As a result, only the posterior halves of the orbits are visible when looked upon from the top and seem to be directed somewhat more laterally. The low occipital margin of the skull determines the relative shallowness of the temporal fossae. On the left side of the skull roof an edge of a bone may be seen, which is wedged between the elements of the anterior border of the orbital ring. This bone apparently presents the nasale which is indicative of a considerable approximation of the nares to the orbit. The ventral surface of the Donguz skull is characterized by shortened and broadened pterygoidea. Choanae are carried somewhat forwards, their anterior edges being slightly driven into the posterior ends of the praemaxillaria. All enumerated features of the structure are characteristic for the genus *Lystrosaurus* to which our form bears the greatest resemblance. The considerable size of the sternum of the skeleton 159/1 PIN is also typical for the *Lystrosaurus*. The long sacrum consisting of eight vertebrae and the correspondingly elongated upper portion of the pelvis with the upper edges of ilia (Fig. 1) approaching one another constitute an entirely distinct affinity of our form.

The gradual increase of the number of sacral vertebrae is very characteristic of the progressive evolutionary development of the most varied groups of animals. The increase of the number of vertebrae is attained through the

inclusion into the sacrum of the presacral vertebrae and leads to the durability of the structure, and the increase of the pelvis area for the attachment of the muscles of the pelvic girdle.

It should be remembered that in man there is also a tendency observable toward the fixation of the hindermost lumbar vertebrae in the sacrum, the so-called sacralization of the vertebral column.

In various forms of the group *Dicynodontidae* we observe the increase of the number of sacral vertebrae in the later forms. Thus, the primitive representatives of the genus *Dicynodon* have altogether four sacral vertebrae, the more advanced having five of them. The largest triassic forms, like, for instance, *Kannemeyeria*, possess a sacrum containing 6 vertebrae.

Fig. 1. *Lystrosaurus klimovi* n. sp. Sacrum from below, $\times 1/4$.

The family *Endothiodontidae*, as a group of primitive forms of *Anomodontia*, are provided with 4 sacral vertebrae.

Representatives of the genus *Dicynodon* (*D. trautscholdi* Am.) of the Northern Dvina pareiasaurian fauna are distinguished by a sacrum consisting of five sacral vertebrae, while it is obvious that the fifth vertebra became only recently part of the sacrum.

The genus *Lystrosaurus* possesses, owing to its later appearance, six sacral vertebrae.

Quite recently F. Huene described from the upper horizons of the middle Trias of Brazil giant specimens of the last *Dicynodont* (skeleton measuring more than 3 m in length) of the genus *Stahleckeria*. In this form the sacrum consists of 8 vertebrae, i. e. the same number as in our form. However, all the vertebrae of the sacrum *Stahleckeria* are fused with one another, while in our form the first three vertebrae are still detached.

In this way, we arrive at the conception of the described form as of a rather progressive representative of the genus *Lystrosaurus*, which has yet not reached the highest degree of specialization and the gigantic height of the last *Anomodontia* from the top of the Middle Trias.

The progressive features of the structure of our form are displayed also through the presence of olecranon ulnae which is practically undeveloped in typical *Dicynodon* and *Lystrosaurus*.

In all probability, a more detailed investigation and description of the available material, rendered more complete by the addition of new collections from the Donguz locality, will reveal some more progressive characters of our form, as compared with the South African *Lystrosaurus*.

On the basis of the aforesaid material, I have no ground for considering the Donguz form so much different from the genus *Lystrosaurus* as to represent a separate genus, and do therefore refer it to *Lystrosaurus*. The progressive characters of our form enumerated above are essential and do not occur in any of the more completely known species of the genus *Lystrosaurus*. It is obvious that our form is at any rate a new species of the genus *Lystrosaurus*, for which I suggest the designation *Lystrosaurus klimovi* n. sp., in honour of P. I. Klimov, the geologist of the Eastern Oil Trust, who had discovered the Donguz locality, as it is due to his efforts that this valuable finding was furnished to the Paleontological Institute undamaged and thus was not lost for science.

The systematic position of the skeleton № 159/1 PIN presents itself as follows: Ordo *Anomodontia*, familia *Dicynodontidae*, genus *Lystrosaurus*, species *klimovi* nov.

Diagnosis of *Lystrosaurus klimovi* n.sp. Skull with low occipital margin, temporal fossae relatively shallow. The length of the skull on the ventral surface is about 400 mm. The parietal crest is short, low, somewhat expanded, a large for. parietale situated on the anterior part of the crest. The frontal portion of the skull roof is broad, praefrontalia forming lateral prominences, overhanging the orbits. Orbits large, with upper border slightly swollen. Articular surfaces of quadrata are on the level of the base of the maxillary tusks. The upper jaw contains 2 large tusks, the rest of the teeth being wanting.

The jugular ribs are double-headed, they begin from the third vertebra. The sacrum consists of eight vertebrae, the upper parts of ilia are strongly elongated antero-posteriorly. The upper edges of ilia are strongly convergent to one another. Ulna has a clearly pronounced olecranon. Total length of the skeleton is about 2 m including skull and tail.

Holotype—skeleton № 158/1 PIN. Locality—right bank of the river Donguz, within the first kilometer below the settlement Petrovskoye, district Sol-Iletzk, Orenburg province, 45 km south-east of Orenburg.

The recovery of *Lystrosaurus klimovi* is the first find of a *Lystrosaurus* within the boundaries of USSR. *Lystrosaurus* is known in South Africa as the typical dominant form of the zone *Lystrosaurus*, strata Beaufort, Karroo formation, extending beyond the limits of Upper Permian and referable to the lowest horizons of the Trias.

Within the territory of the USSR no triassic deposits have been known heretofore with a well definable reptilian fauna.

In view of the fact that *Lystrosaurus klimovi* is more highly organized than the typical species *Lystrosaurus* of South Africa, it should characterize a higher horizon of the Trias. The comparison of *Lystrosaurus klimovi* with the giant *Stahleckeria* of the Middle Trias of Brazil emphasizes the greater primitiveness of our form. As against *Stahleckeria* the age of *L. klimovi* should be the more ancient.

Most probably, the Donguz form characterizes the upper regions of the continental Lower Trias—the zone of *Cynognathus* of South Africa, corresponding to the German upper variegated sandstone (Bunter), or the Scythian strata of marine Trias.

The deposits of that age, differentiated by me into the VI faunistic zone, have been known only in the mountain Bolshaya Bogdo near the lake Baskunchak in the Lower Volga region and characterized solely by labyrinthodontia. The finding of a new form *Lystrosaurus* in the Orenburg province establishes unmistakably the triassic age of the variegated deposits of the Donguz district. In all probability, further research will show a wide distribution of triassic deposits in the South Ural region.

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